Advanced Math—Exponents and Roots

Exponent Rules

Memorize these exponent rules that are frequently tested on the SAT Math:

Exponent Rule	Example	How to Remember
$x^a x^b = x^{(a+b)}$	$x^2 x^3 = x^{(2+3)} = x^5$	*Use the acronym **MADSPM*** **M**ultiply exponents, **A**dd them **D**ivide exponents, **S**ubtract them **P**arentheses with exponents, **M**ultiply them
$\frac{x^a}{x^b} = x^{a-b}$	$\frac{x^5}{x^2} = x^{5-2} = x^3$	
$(x^a)^b = x^{(ab)}$	$(x^2)^3 = x^{(2\times 3)} = x^6$	
$x^{-a} = \frac{1}{x^a}$	$x^{-3} = \frac{1}{x^3}$	"Negative" things go to the bottom.
$x^{\frac{a}{b}} = \sqrt[b]{x^a}$	$x^{\frac{2}{3}} = \sqrt[3]{x^2}$	The root of a tree is on the bottom. The bottom part of the fraction is identical to the root.

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Practice

Write these expressions in simplified, alternative forms.

1. $a^2 a^6 = ?$
2. $\frac{x^7}{x^4} = ?$
3. $y^{(\frac{3}{4})} = ?$
4. $b^{-4} = ?$
5. $(n^3)^4 = ?$
6. $(x^2 x^3)^2 = ?$
7. $\frac{1}{a^{-3}} = ?$
8. $\sqrt[3]{216x^{12}} = ?$

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Problems with Exponents and Roots

SAT Math problems that involve exponents and roots will often ask you to integrate multiple concepts to arrive at a solution.

› Example

If $x^{(-\frac{1}{3})} = y$ and x is positive, what is x in terms of y?

(A) $y = x^3$

(B) $y = \sqrt[3]{x}$

(C) $x = \frac{1}{y^3}$

(D) $x = y^3$

✓ Solution

When a problem asks you to put x in terms of y, it means that you should have x by itself and an expression in terms of y on the other side. So, we should isolate the x variable to arrive at a solution:

$$x^{(-\frac{1}{3})} = y \rightarrow$$

Take each side to the -3 power to get a simple $x \rightarrow$

$$\left(x^{(-\frac{1}{3})}\right)^{(-3)} = y^{-3} \rightarrow$$

$$x^{(-\frac{1}{3} \times -3)} = y^{-3} \rightarrow$$

$$x^1 = y^{-3} \rightarrow$$

$$x = \frac{1}{y^3}$$

SAT Math Strategy

Memorize the values of simple numbers to different powers to more easily recognize patterns.

The SAT Math heavily emphasizes pattern recognition and does *not* emphasize tedious calculations. As a result, you will frequently find that problems involving exponents and roots will involve simple numbers and their exponential values and roots. Memorizing these equivalences will save you time.

$x^0 = 1$ Anything to the zero power is simply one. $1^0 = 1\ 2^0 = 1\ 10^0 = 1$

$2^1 = 2$ Anything to the 1st power is simply itself. $3^1 = 3\ 20^1 = 20\ (-5)^1 = -5$

$2^2 = 4 \quad 2^3 = 8 \quad 2^4 = 16 \quad 2^5 = 32 \quad 2^6 = 64$

$\sqrt{4} = 2 \quad \sqrt[3]{8} = 2 \quad \sqrt[4]{16} = 2 \quad \sqrt[5]{32} = 2 \quad \sqrt[6]{64} = 2$

$3^2 = 9 \quad 3^3 = 27 \quad 3^4 = 81$

$\sqrt{9} = 3 \quad \sqrt[3]{27} = 3 \quad \sqrt[4]{81} = 3$

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$4^2 = 16 \quad 4^3 = 64 \quad 4^4 = 256$

$\sqrt{16} = 4 \quad \sqrt[3]{64} = 4 \quad \sqrt[4]{256} = 4$

$5^2 = 25 \quad 5^3 = 125$

$\sqrt{25} = 5 \quad \sqrt[3]{125} = 5$

$6^2 = 36 \quad 6^3 = 216$

$\sqrt{36} = 6 \quad \sqrt[3]{216} = 6$

$7^2 = 49 \quad 8^2 = 64 \quad 9^2 = 81$

$\sqrt{49} = 7 \quad \sqrt{64} = 8 \quad \sqrt{81} = 9$

$10^2 = 100 \quad 10^3 = 1{,}000 \quad 10^4 = 10{,}000$

$\sqrt{100} = 10 \quad \sqrt[3]{1{,}000} = 10 \quad \sqrt[4]{10{,}000} = 10$

Example 1

If $2^{(x-5)} = 8$, what is the value of x?

(A) 4
(B) 5
(C) 7
(D) 8

Solution

Use your knowledge of the exponential values of 2 to see a shortcut. Since $2^3 = 8$, simply set the exponent of 2 equal to 3 to solve for x:

$$x - 5 = 3 \rightarrow x = 8$$

So, choice **(D)** is correct. Alternatively, you could work your way backwards from the answer choices. When you plug in choice (D), you will also arrive at the correct answer:

$$2^{(x-5)} = 8 \rightarrow \text{Plug in 8 for } x \rightarrow$$
$$2^{(8-5)} = 8 \rightarrow 2^3 = 8$$

Your knowledge of the exponential values of 2 will also help you quickly see the correct answer when you solve by plugging in the answer choices.

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Example 2

x	$f(x)$
1	1
2	4
3	9
4	16

In the table above, which of the following defines the function $f(x)$?

(A) $f(x) = x + 3$
(B) $f(x) = x^2$
(C) $f(x) = -x^2$
(D) $f(x) = x^3$

Solution

Use your knowledge of the exponential values of simple numbers to recognize the pattern.

$$1^2 = 1$$
$$2^2 = 2$$
$$3^2 = 9$$
$$4^2 = 16$$

Choice **(B)**, $f(x) = x^2$, therefore correctly models this relationship.

Practice Drill—Exponents and Roots

(If timing, take about 12 minutes to complete.)

1. Which of these expressions is equivalent to $\sqrt[4]{x^3}$?

   (A) $x^{\left(\frac{4}{3}\right)}$
   (B) $\frac{x^3}{x^4}$
   (C) $\frac{x^4}{x^3}$
   (D) $x^{\left(\frac{3}{4}\right)}$

2. If $3x^2 - 27 = 0$, and $x > 0$, what is the value of x?

   (A) 1
   (B) 2
   (C) 3
   (D) 4

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3. $f(x) = \frac{5^x}{2^x}$

What is the value of $f(2)$ in the function above?

(A) $\frac{25}{4}$

(B) $\frac{5}{2}$

(C) $\frac{10}{4}$

(D) 8

4. If $f(x) = \frac{\sqrt{x}}{x}$, what is $f(9)$?

(A) $\frac{1}{9}$

(B) $\frac{1}{3}$

(C) 1

(D) 3

5. If x is positive, which expression is equivalent to $\sqrt[3]{27x^6}$?

(A) $3x$

(B) $3x^2$

(C) $6x^2$

(D) $9x^3$

x	f(x)
1	2
2	5
3	10
4	17
5	26

6. Based on the values in the table above, which of the following correctly expresses the value of $f(x)$?

(A) $f(x) = x + 1$

(B) $f(x) = x^2$

(C) $f(x) = x^3$

(D) $f(x) = x^2 + 1$

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7. $\frac{x^a y^{-b}}{x^{-a} y^b}$

Consider the expression above, in which x and y are both positive, and a and b are positive constants. Which of the following would be an equivalent form of the expression?

(A) $\frac{x^a}{y^b}$

(B) $\frac{x^{(a^2)}}{y^{(b^2)}}$

(C) $\frac{x^{2a}}{y^{2b}}$

(D) $\frac{y}{x}$

Fill-In Practice: Write your answer in the underlined blank under each question.

8. $\sqrt[3]{125} + \sqrt[3]{64} = x^2$

Given that $x > 0$, what is the value of x in the above equation?

Answer: _____

9. If $x^{-2} y^{-1} = \frac{1}{5}$, what is the value of $x^4 y^2$?

Answer: _____

10. If $\sqrt{\frac{x}{2}} = 4$, what is $\sqrt[5]{x}$?

Answer: _____

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10. **(2)**

$\sqrt{\frac{x}{2}} = 4 \rightarrow$

Square both sides $\rightarrow$

$\frac{x}{2} = 16 \rightarrow$

$x = 32$

Take the fifth root of 32 $\rightarrow$

$\sqrt[5]{32} = 2$

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