Advanced Math—Exponents and Roots

Exponent Rules

Memorize these exponent rules that are frequently tested on the SAT Math:

Exponent Rule	Example	How to Remember
$x^a x^b = x^{(a+b)}$	$x^2 x^3 = x^{(2+3)} = x^5$	*Use the acronym **MADSPM*** **M**ultiply exponents, **A**dd them **D**ivide exponents, **S**ubtract them **P**arentheses with exponents, **M**ultiply them
$\frac{x^a}{x^b} = x^{a-b}$	$\frac{x^5}{x^2} = x^{5-2} = x^3$	
$(x^a)^b = x^{(ab)}$	$(x^2)^3 = x^{(2\times 3)} = x^6$	
$x^{-a} = \frac{1}{x^a}$	$x^{-3} = \frac{1}{x^3}$	"Negative" things go to the bottom.
$x^{\frac{a}{b}} = \sqrt[b]{x^a}$	$x^{\frac{2}{3}} = \sqrt[3]{x^2}$	The root of a tree is on the bottom. The bottom part of the fraction is identical to the root.

$y^{\frac{3}{4}} = \sqrt[4]{y^3}$

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Practice

Write these expressions in simplified, alternative forms.

1. $a^2 a^6 = ?$ a^8
2. $\frac{x^7}{x^4} = ?$ x^3
3. $y^{(\frac{3}{4})} = ?$
4. $b^{-4} = ?$ $\frac{1}{b^4}$
5. $(n^3)^4 = ?$ n^{12}
6. $(x^2 x^3)^2 = ?$ $x^4 x^6 = x^{10}$
7. $\frac{1}{a^{-3}} = ?$ $a^{-(-3)} = a^3$
8. $\sqrt[3]{216x^{12}} = ?$ $(216)^{\frac{1}{3}} x^{\frac{12}{3}} = 6x^4$

$216 = 6^3$ $(216)^{\frac{1}{3}} = (6^3)^{\frac{1}{3}} = 6$

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Problems with Exponents and Roots

SAT Math problems that involve exponents and roots will often ask you to integrate multiple concepts to arrive at a solution.

Example

If $x^{(-\frac{1}{3})} = y$ and x is positive, what is x in terms of y?

(A) $y = x^3$
(B) $y = \sqrt[3]{x}$
(C) $x = \frac{1}{y^3}$
(D) $x = y^3$

Solution

When a problem asks you to put x in terms of y, it means that you should have x by itself and an expression in terms of y on the other side. So, we should isolate the x variable to arrive at a solution:

$$x^{(-\frac{1}{3})} = y \rightarrow$$

Take each side to the -3 power to get a simple $x \rightarrow$

$$\left(x^{(-\frac{1}{3})}\right)^{(-3)} = y^{-3} \rightarrow$$

$$x^{(-\frac{1}{3} \times -3)} = y^{-3} \rightarrow$$

$$x^1 = y^{-3} \rightarrow$$

$$x = \frac{1}{y^3}$$

SAT Math Strategy

Memorize the values of simple numbers to different powers to more easily recognize patterns.

The SAT Math heavily emphasizes pattern recognition and does *not* emphasize tedious calculations. As a result, you will frequently find that problems involving exponents and roots will involve simple numbers and their exponential values and roots. Memorizing these equivalences will save you time.

$x^0 = 1$ Anything to the zero power is simply one. $1^0 = 1\ 2^0 = 1\ 10^0 = 1$

$2^1 = 2$ Anything to the 1st power is simply itself. $3^1 = 3\ 20^1 = 20\ (-5)^1 = -5$

$2^2 = 4 \quad 2^3 = 8 \quad 2^4 = 16 \quad 2^5 = 32 \quad 2^6 = 64$
$\sqrt{4} = 2 \quad \sqrt[3]{8} = 2 \quad \sqrt[4]{16} = 2 \quad \sqrt[5]{32} = 2 \quad \sqrt[6]{64} = 2$

$3^2 = 9 \quad 3^3 = 27 \quad 3^4 = 81$
$\sqrt{9} = 3 \quad \sqrt[3]{27} = 3 \quad \sqrt[4]{81} = 3$

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$4^2 = 16 \quad 4^3 = 64 \quad 4^4 = 256$

$\sqrt{16} = 4 \quad \sqrt[3]{64} = 4 \quad \sqrt[4]{256} = 4$

$5^2 = 25 \quad 5^3 = 125$

$\sqrt{25} = 5 \quad \sqrt[3]{125} = 5$

$6^2 = 36 \quad 6^3 = 216$

$\sqrt{36} = 6 \quad \sqrt[3]{216} = 6$

$7^2 = 49 \quad 8^2 = 64 \quad 9^2 = 81$

$\sqrt{49} = 7 \quad \sqrt{64} = 8 \quad \sqrt{81} = 9$

$10^2 = 100 \quad 10^3 = 1,000 \quad 10^4 = 10,000$

$\sqrt{100} = 10 \quad \sqrt[3]{1,000} = 10 \quad \sqrt[4]{10,000} = 10$

Example 1

If $2^{(x-5)} = 8$, what is the value of x?

(A) 4
(B) 5
(C) 7
(D) 8

Solution

Use your knowledge of the exponential values of 2 to see a shortcut. Since $2^3 = 8$, simply set the exponent of 2 equal to 3 to solve for x:

$$x - 5 = 3 \rightarrow x = 8$$

So, choice **(D)** is correct. Alternatively, you could work your way backwards from the answer choices. When you plug in choice (D), you will also arrive at the correct answer:

$$2^{(x-5)} = 8 \rightarrow \text{Plug in 8 for } x \rightarrow$$
$$2^{(8-5)} = 8 \rightarrow 2^3 = 8$$

Your knowledge of the exponential values of 2 will also help you quickly see the correct answer when you solve by plugging in the answer choices.

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Example 2

x	f(x)
1	1
2	4
3	9
4	16

In the table above, which of the following defines the function $f(x)$?

(A) $f(x) = x + 3$
(B) $f(x) = x^2$
(C) $f(x) = -x^2$
(D) $f(x) = x^3$

Solution

Use your knowledge of the exponential values of simple numbers to recognize the pattern.

$$1^2 = 1$$
$$2^2 = 2$$
$$3^2 = 9$$
$$4^2 = 16$$

Choice **(B)**, $f(x) = x^2$, therefore correctly models this relationship.

Practice Drill—Exponents and Roots

(If timing, take about 12 minutes to complete.)

1. Which of these expressions is equivalent to $\sqrt[4]{x^3}$?

   (A) $x^{\left(\frac{4}{3}\right)}$
   (B) $\frac{x^3}{x^4}$
   (C) $\frac{x^4}{x^3}$
   (D) $x^{\left(\frac{3}{4}\right)}$

2. If $3x^2 - 27 = 0$, and $x > 0$, what is the value of x?

   (A) 1
   (B) 2
   (C) 3
   (D) 4

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3. $f(x) = \frac{5^x}{2^x}$

What is the value of $f(2)$ in the function above?

(A) $\frac{25}{4}$

(B) $\frac{5}{2}$

(C) $\frac{10}{4}$

(D) 8

4. If $f(x) = \frac{\sqrt{x}}{x}$, what is $f(9)$?

(A) $\frac{1}{9}$

(B) $\frac{1}{3}$

(C) 1

(D) 3

5. If x is positive, which expression is equivalent to $\sqrt[3]{27x^6}$?

(A) $3x$

(B) $3x^2$

(C) $6x^2$

(D) $9x^3$

x	f(x)
1	2
2	5
3	10
4	17
5	26

6. Based on the values in the table above, which of the following correctly expresses the value of $f(x)$?

(A) $f(x) = x + 1$

(B) $f(x) = x^2$

(C) $f(x) = x^3$

(D) $f(x) = x^2 + 1$

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7. $\frac{x^a y^{-b}}{x^{-a} y^b}$

Consider the expression above, in which x and y are both positive, and a and b are positive constants. Which of the following would be an equivalent form of the expression?

(A) $\frac{x^a}{y^b}$

(B) $\frac{x^{(a^2)}}{y^{(b^2)}}$

(C) $\frac{x^{2a}}{y^{2b}}$

(D) $\frac{y}{x}$

$\frac{x^a}{x^{-a}} \cdot \frac{y^{-b}}{y^b}$

Fill-In Practice: Write your answer in the underlined blank under each question.

8. $\sqrt[3]{125} + \sqrt[3]{64} = x^2$

Given that $x > 0$, what is the value of x in the above equation?

Answer: 3

9. If $x^{-2}y^{-1} = \frac{1}{5}$, what is the value of x^4y^2?

Answer: 25

$x^{-2}y^{-1} = \frac{1}{x^2 y} = \frac{1}{5}$

$x^2 y = 5$

$x^4 y^2 = 5^2 = 25$

10. If $\sqrt{\frac{x}{2}} = 4$, what is $\sqrt[5]{x}$?

Answer: _____

$\sqrt{\frac{x}{2}} = 4$ $\left(\left(\frac{x}{2}\right)^{\frac{1}{2}}\right)^2 = \frac{x}{2} = 4^2 = 16$

$x = 32$ $\sqrt[5]{32} = 32^{\frac{1}{5}} = 2$

$2^2 = 4$ $2^3 = 8$ $2^4 = 16$ $2^5 = 32$

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10. **(2)**

$\sqrt{\frac{x}{2}} = 4 \rightarrow$

Square both sides $\rightarrow$

$\frac{x}{2} = 16 \rightarrow$

$x = 32$

Take the fifth root of 32 $\rightarrow$

$\sqrt[5]{32} = 2$

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EXERCISE 1: Evaluate WITHOUT a calculator. Answers for this exercise start on page 311.

1. $(-1)^{10}$
2. $(-1)^{15}$
3. $(-1)^{8}$
4. -1^{8}
5. $-(-1)^{8}$
6. $(-3)^{3}$
7. -3^{3}
8. $-(-6)^{2}$
9. $-(-4)^{3}$
10. $2^{3} \times 3^{2} \times (-1)^{5}$
11. 4^{-1}
12. 5^{0}
13. 3^{-2}
14. $\left(\frac{1}{2}\right)^{-1}$
15. $\left(\frac{4}{3}\right)^{-2}$

EXERCISE 2: Simplify so that your answer contains only positive exponents. The first one has been done for you. Answers for this exercise start on page 312.

1. $2k^{-4} \cdot 4k^{2} = \frac{2 \cdot 4k^{2}}{k^{4}} = \frac{8}{k^{2}}$
2. $3x^{2} \cdot 2x^{3}$
3. $5x^{4} \cdot 3x^{-2}$
4. $7m^{3} \cdot -3m^{-3}$
5. $(2x^{2})^{-3}$
6. $-3a^{2}b^{-3} \cdot 3a^{-5}b^{8}$
7. $\frac{3x^{4}}{(x^{-2})^{2}}$ $3x^{8}$
8. $\frac{x^{\frac{3}{2}}}{x^{\frac{1}{2}}}$
9. $(2m)^{2} \cdot (3m^{3})^{2}$
10. $(a^{-1} \cdot a^{-2})^{2}$
11. $(b^{-2})^{-3} \cdot (b^{3})^{2}$ $b^{0} = 1$
12. $\frac{(m^{2}n)^{3}}{(mn^{2})^{2}}$ $n^{0} = 1$
13. $\frac{mn}{m^{2}n^{3}}$
14. $\frac{k^{-2}}{k^{-3}}$
15. $\frac{x^{2}y^{3}z^{4}}{x^{-3}y^{-4}z^{-5}}$

EXERCISE 3: Simplify the radicals or solve for x. Answers for this exercise start on page 312.

1. $\sqrt{12}$
2. $\sqrt{96}$
3. $\sqrt{45}$
4. $\sqrt{18}$
5. $3\sqrt{75}$
6. $\sqrt{32}$
7. $5\sqrt{2} = \sqrt{x}$
8. $3\sqrt{x} = \sqrt{45}$
9. $2\sqrt{2} = \sqrt{4x}$
10. $4\sqrt{6} = 2\sqrt{3x}$
11. $3\sqrt{8} = x\sqrt{2}$
12. $x\sqrt{x} = \sqrt{216}$

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EXERCISE 4: Answers for this exercise start on page 313.

1

Which expression is equivalent to $(x^2y)(x^4y^{-3})$, where x, y, and z are positive numbers?

A) x^6y^{-3}
B) x^6y^{-2}
C) x^8y^{-3}
D) x^8y^{-2}

2

Which of the following is equivalent to $\sqrt{\frac{x}{64}}$ for all $x > 0$?

A) $\frac{x^2}{8}$
B) $\frac{x^2}{32}$
C) $\frac{x^{\frac{1}{2}}}{8}$
D) $\frac{x^{\frac{1}{2}}}{32}$

3

Which expression is equivalent to $\frac{x^{\frac{5}{6}}}{\sqrt[3]{x}}$, where $x \neq 0$?

A) $x^{\frac{1}{2}}$
B) $x^{\frac{5}{2}}$
C) $x^{\frac{2}{3}}$
D) $x^{\frac{5}{3}}$

4

Which expression is equivalent to $b^{\frac{9}{7}}$, where $b > 0$?

A) $\sqrt[15]{b^{105}}$
B) $\sqrt[15]{b^{135}}$
C) $\sqrt[105]{b^{135}}$
D) $\sqrt[135]{b^{105}}$

5

Which expression represents the product of $\left(b^6c^{-2}d^{-5}\right)$ and $\left(b^8c^{-3}+c^4d^5\right)$, where b, c, and d are positive?

A) $b^{14}c^{-5}+c^2d^{-10}$
B) $b^{14}c^{-5}+c^2$
C) $b^{14}c^{-5}d^{-5}+b^6c^2$
D) $b^{14}c^{-5}d^{-5}+c^2$

6

If $x \neq 0$ and $y \neq 0$, which of the following is equivalent to $\frac{8x^2}{\sqrt{4x^6y^4}}$?

A) $2xy^{-2}$
B) $4x^{-2}y^2$
C) $4x^{-1}y^{-2}$
D) $4xy^2$

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$$(b^6 c^{-2} d^{-5})(b^8 c^{-3} + c^4 d^5)$$

$$(b^6 c^{-2} d^{-5})(b^8 c^{-3}) + (b^6 c^{-2} d^{-5})(c^4 d^5)$$

$$b^{6+8} c^{-2-3} d^{-5} = b^{14} c^{-5} d^{-5}$$

$$b^6 c^{-2+4} d^{-5+5} = b^6 c^2 d^0 = b^6 c^2$$

7

If r and s are positive, which of the following expressions is equivalent to $r^{\frac{6}{7}}s^{\frac{3}{7}}$?

A) $\frac{1}{\sqrt[6]{r^7s^{14}}}$

B) $\sqrt[6]{r^7s^{14}}$

C) $\frac{1}{\sqrt[7]{r^6s^3}}$

D) $\sqrt[7]{r^6s^3}$

8

$$\sqrt[b]{a} = \sqrt[d]{c}$$

The given equation relates the distinct positive numbers a, b, c, and d. Which equation correctly expresses c in terms of a, b, and d ?

A) $c = \frac{d}{a^{\frac{1}{b}}}$

B) $c = a^{d-b}$

C) $c = a^{\frac{b}{d}}$

D) $c = a^{\frac{d}{b}}$

9

Which expression is equivalent to $\sqrt{16a^{\frac{2}{3}}}$, where $a > 0$?

A) $4a^{\frac{1}{3}}$

B) $4a^{\frac{4}{3}}$

C) $8a^{\frac{1}{3}}$

D) $8a^{\frac{4}{3}}$

10

Which expression is equivalent to $\sqrt{rs}(\sqrt{r}+\sqrt{s})$, where $r \geq 0$ and $s \geq 0$?

A) $\sqrt{r^2s+rs^2}$

B) $r\sqrt{s}+s\sqrt{r}$

C) $rs\sqrt{r+s}$

D) $\sqrt{rs+r+s}$

11

The expression $\left(\sqrt[3]{x^2}\right)^n$, where n is a constant, is equivalent to x^8. What is the value of n ?

$n = 12$

12

Which expression is equivalent to $2^{-2k} \cdot 3^k$?

A) $(3\sqrt{2})^k$

B) $\left(\frac{1}{36}\right)^k$

C) $\left(\frac{3}{4}\right)^k$

D) $\left(\frac{9}{4}\right)^k$

$2^{-2k} = \frac{1}{2^{2k}} = \frac{1}{(2^2)^k} = \frac{1}{4^k} = \left(\frac{1}{4}\right)^k$

$\left(\frac{1}{4}\right)^k \cdot 3^k = \left(\frac{3}{4}\right)^k$

13

Which expression is equivalent to $a^{\frac{2}{9}}(a^{\frac{2}{3}})^{\frac{2}{3}}$, where a is positive?

A) $\sqrt[3]{a^2}$

B) $\sqrt[3]{a^{16}}$

C) $\sqrt[9]{a^8}$

D) $\sqrt[9]{a^{14}}$

$\left(a^{\frac{2}{3}}\right)^{\frac{2}{3}} = a^{\frac{4}{9}}$

$a^{\frac{2}{9}} \cdot a^{\frac{4}{9}} = a^{\frac{6}{9}}$

$= a^{\frac{2}{3}} = \sqrt[3]{a^2}$

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14

Which expression is equivalent to $\frac{4}{9}\left(\frac{2}{3}\right)^a$?

A) $\left(\frac{2}{3}\right)^{\frac{a}{2}}$

B) $\left(\frac{2}{3}\right)^{a-2}$

C) $\left(\frac{2}{3}\right)^{a+2}$

D) $\left(\frac{2}{3}\right)^{2a}$

15

$$\sqrt[4]{x^3} \cdot \sqrt[12]{x^5}$$

The given expression is equivalent to $\sqrt[m]{x^n}$, where m and n are positive constants each less than 10 and $x > 1$. What is the value of $\frac{m}{n}$?

$\sqrt[m]{x^n} = x^{\frac{n}{m}} = x^{\frac{7}{6}}$

$\frac{n}{m} = \frac{7}{6}$ $\frac{m}{n} = \frac{6}{7}$

$\sqrt[4]{x^3} = x^{\frac{3}{4}}$ $x^{\frac{3}{4}} \cdot x^{\frac{5}{12}}$

$\sqrt[12]{x^5} = x^{\frac{5}{12}}$

$\frac{3}{4} + \frac{5}{12} = \frac{3\times3+5}{12} = \frac{14}{12} = \frac{7}{6}$

16

Two numbers, p and q, are each greater than zero, and the square of p is equal to the cube root of q. For what value of x is p^{1-x} equal to the square root of q ?

$p^2 = \sqrt[3]{q^6} = q^{\frac{1}{3}}$

$\sqrt{p^6} = \sqrt{q^6}$

$(p^2)^3 = \left(q^{\frac{1}{3}}\right)^3$

$p^{\frac{6}{2}} = q^{\frac{1}{2}}$

$p^6 = q^6$

$p^3 = q^{\frac{1}{2}} = p^{1-x}$

$3 = 1-x$

$3+x=1$ $x = 1-3 = -2$

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