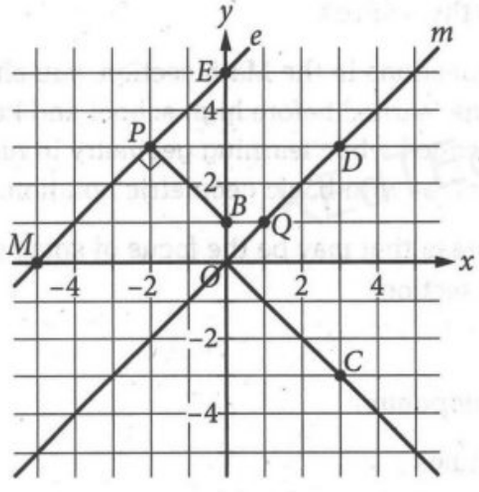
REMINDER

The triangle inequality theorem states that for any triangle, the length of any side of the triangle must be less than the sum of the lengths of the other two sides of the triangle and greater than the difference of the lengths of the other two sides.

- Triangles and other polygons
  - Right triangles and the Pythagorean theorem
  - Properties of equilateral and isosceles triangles
  - Properties of 30°-60°-90° triangles and 45°-45°-90° triangles
  - Congruent triangles and other congruent figures
  - Similar triangles and other similar figures
  - The triangle inequality theorem
  - Squares, rectangles, parallelograms, trapezoids, and other quadrilaterals
  - Regular polygons
- Circles
  - Radius, diameter, and circumference
  - Measure of central angles and inscribed angles
  - Arc length, arc measure, and area of sectors
  - Tangents and chords
  - Representations in the *xy*-plane
- Area and volume
  - Area of plane figures
  - Volume of solids
  - Surface area of solids

You should be familiar with the geometric notation for points and lines, line segments, angles and their measures, and lengths.



In the figure shown, the *xy*-plane has origin *O*. The values of *x* on the horizontal *x*-axis increase as you move to the right, and the values of *y* on the vertical *y*-axis increase as you move up. Line *e* contains point *P*, which has coordinates (−2, 3); point *E*, which has coordinates (0, 5); and point *M*, which has coordinates (−5, 0). Line *m* passes through the origin *O* (0, 0), the point *Q* (1, 1), and the point *D* (3, 3).

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Lines e and m are parallel—they never meet. This is written $e \parallel m$.

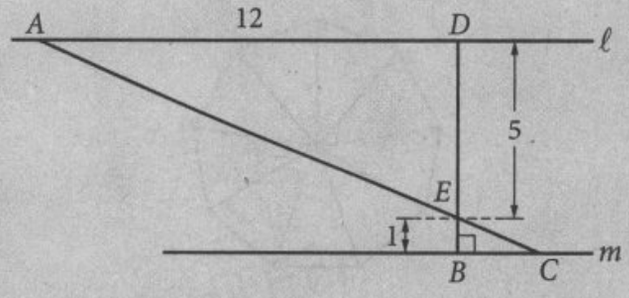
You'll also need to know the following notation:

- $\overleftrightarrow{PE}$: The line containing the points P and E (this is the same as line e)
- $\overline{PE}$, or line segment PE: The line segment with endpoints P and E
- PE: The length of segment PE (you can write $PE = 2\sqrt{2}$)
- $\overrightarrow{PE}$: The ray starting at point P and extending indefinitely in the direction of point E
- $\overrightarrow{EP}$: The ray starting at point E and extending indefinitely in the direction of point P
- $\angle DOC$: The angle formed by $\overrightarrow{OD}$ and $\overrightarrow{OC}$
- $\triangle PEB$: The triangle with vertices P, E, and B
- Quadrilateral $BPMO$: The quadrilateral with vertices B, P, M, and O
- $\overline{BP} \perp \overline{PM}$: Segment BP is perpendicular to segment PM. (You should also recognize that the right-angle box within $\angle BPM$ means this angle is a right angle.)

STRATEGY

Familiarize yourself with these notations in order to avoid confusion on test day.

Geometry and Trigonometry Sample Question 1



In the figure shown, line ℓ is parallel to line m, segment BD is perpendicular to line m, and segment AC and segment BD intersect at E. What is the length of segment AC?

Note that this sample question has no answer choices. It's in the student-produced response (SPR) format. You'll have to come up with your own answer.

Since segment AC and segment BD intersect at E, $\angle AED$ and $\angle CEB$ are vertical angles, and so the measure of $\angle AED$ is equal to the measure of $\angle CEB$. Since line ℓ is parallel to line m, $\angle BCE$ and $\angle DAE$ are alternate interior angles of parallel lines cut by a transversal, and so the measure of $\angle BCE$ is equal to the measure of $\angle DAE$. By the angle-angle similarity theorem, $\triangle AED$ is similar to $\triangle CEB$, with vertices A, E, and D corresponding to vertices C, E, and B, respectively.

Also, $\triangle AED$ is a right triangle, so by the Pythagorean theorem, $AE = \sqrt{AD^2 + DE^2}$. Substituting values for the length of the line segments gives $\sqrt{12^2 + 5^2} = \sqrt{169}$, which means that $AE = 13$.

STRATEGY

A shortcut here is remembering that 5, 12, 13 is a Pythagorean triple (5 and 12 are the lengths of the sides of the right triangle, and 13 is the length of the hypotenuse). Another common Pythagorean triple is 3, 4, 5.

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Since $\triangle AED$ is similar to $\triangle CEB$, the ratios of the lengths of corresponding sides of the two triangles are in the same proportion. Since $\frac{ED}{EB} = \frac{AE}{EC}$, then $\frac{5}{1} = \frac{13}{EC}$, and so $EC = \frac{13}{5}$. Therefore, $AC = AE + EC$, or $13 + \frac{13}{5} = \frac{78}{5}$.

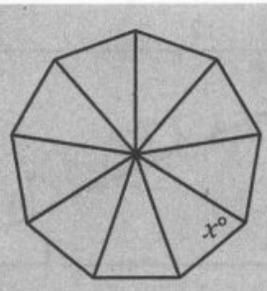
Note some of the key concepts that were used in sample 1:

- Vertical angles have the same measure.
- When parallel lines are cut by a transversal, the alternate interior angles have the same measure.
- If two angles of a triangle are congruent to (have the same measure as) two angles of another triangle, the two triangles are similar.
- The Pythagorean theorem: $a^2 + b^2 = c^2$, where a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse.
- If two triangles are similar, then all ratios of lengths of corresponding sides are equal.
- If point E lies on line segment AC, then $AC = AE + EC$.

STRATEGY

Note how sample 1 requires the knowledge and application of numerous fundamental geometry concepts. Develop mastery of the fundamental concepts, and practice applying them on test-like questions.

Geometry and Trigonometry Sample Question 2



In the figure shown, a regular polygon with 9 sides has been divided into 9 congruent isosceles triangles by line segments drawn from the center of the polygon to its vertices. What is the value of x?

The sum of the measures of the angles around a point is 360°. Since the 9 triangles are congruent, the measures of each of the 9 angles around the center point are equal. Thus, the measure of each of the 9 angles around the center point is $\frac{360°}{9} = 40°$. In any triangle, the sum of the measures of the interior angles is 180°. So in each triangle, the sum of the measures of the remaining two angles is $180° - 40° = 140°$. Since each triangle is isosceles, the measure of each of these two angles is the same. Therefore, the measure of each of these angles is $\frac{140°}{2} = 70°$. Hence, the value of x is 70.

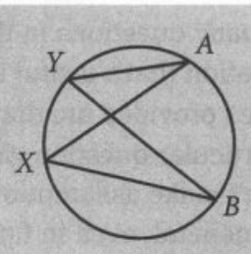
Note some of the key concepts that were used in sample 2:

- The sum of the measures of the angles about a point is 360°.
- Corresponding angles of congruent triangles have the same measure.

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- The sum of the measures of the interior angles of any triangle is 180°.
- In an isosceles triangle, the angles opposite the sides of equal length are of equal measure.

Geometry and Trigonometry Sample Question 3



In the figure shown, $\angle AXB$ and $\angle AYB$ are inscribed in the circle. Which of the following statements is true?

A) The measure of $\angle AXB$ is greater than the measure of $\angle AYB$.
B) The measure of $\angle AXB$ is less than the measure of $\angle AYB$.
C) The measure of $\angle AXB$ is equal to the measure of $\angle AYB$.
D) There is not enough information to determine the relationship between the measure of $\angle AXB$ and the measure of $\angle AYB$.

Choice C is correct. Let the measure of arc AB be $d°$. Since $\angle AXB$ is inscribed in the circle and intercepts arc AB, the measure of $\angle AXB$ is equal to half the measure of arc AB. Thus, the measure of $\angle AXB$ is $\frac{d°}{2}$. Similarly, since $\angle AYB$ is also inscribed in the circle and intercepts arc AB, the measure of $\angle AYB$ is also $\frac{d°}{2}$. Therefore, the measure of $\angle AXB$ is equal to the measure of $\angle AYB$.

STRATEGY

At first glance, it may appear as though there's not enough information to determine the relationship between the two angle measures. One key to this question is identifying what's the same about the two angle measures. In this case, both angles intercept arc AB.

Note the key concept that was used in sample 3:

- The measure of an angle inscribed in a circle is equal to half the measure of its intercepted arc.

You also should know these related concepts:

- The measure of a central angle in a circle is equal to the measure of its intercepted arc.
- An arc is measured in degrees, while arc length is measured in linear units.

You should also be familiar with notation for arcs and circles for questions in the Math section.

- A circle may be identified by the point at its center—for instance, "the circle centered at point M" or "the circle with center at point M."
- An arc named with only its two endpoints, such as $\overset{\frown}{AB}$ or arc AB, will always refer to a minor arc. A minor arc has a measure that's less than 180°.

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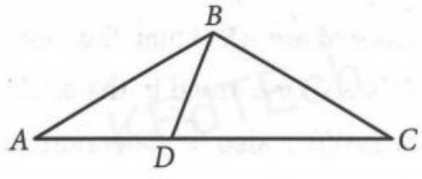
- An arc may also be named with three points: the two endpoints and a third point that the arc passes through. So $\overset{\frown}{ACB}$ or arc *ACB* has endpoints at *A* and *B* and passes through point *C*. Three points may be used to name a minor arc or an arc that has a measure of 180° or more.

REMINDER

Figures are drawn to scale in the SAT Math section unless explicitly stated otherwise. If a question states that a figure isn't drawn to scale, be careful not to make unwarranted assumptions about the figure.

In general, figures that accompany questions in the Math section are intended to provide information that's useful in answering the question. Sometimes figures provided are drawn to scale, and sometimes they're not. In a particular question when it's stated that the figure isn't drawn to scale, don't make assumptions about the relative size of angles or segments. In general, even in figures not drawn to scale, the relative positions of points and angles may be assumed to be in the order shown. Also, line segments that extend through points and appear to lie on the same line may be assumed to be on the same line. A point that appears to lie on a line or curve may be assumed to lie on the line or curve.

The text "Note: Figure not drawn to scale" is included with the figure when degree measures may not be accurately shown and specific lengths may not be drawn proportionally. The following example illustrates what information can and can't be assumed from a figure not drawn to scale.



Note: Figure not drawn to scale.

A question may refer to a triangle such as *ABC* above. Although the note indicates that the figure isn't drawn to scale, you may assume the following from the figure:

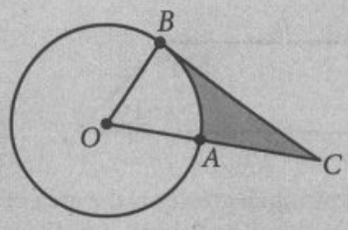
- *ABC*, *ABD*, and *DBC* are triangles.
- *A*, *D*, and *C* are points on a line.
- *D* is between *A* and *C*.
- The length of $\overline{AD}$ is less than the length of $\overline{AC}$.
- The measure of angle *ABD* is less than the measure of angle *ABC*.

You may *not* assume the following from the figure:

- The length of $\overline{AD}$ is less than the length of $\overline{DC}$.
- The measures of angles *BAD* and *DBA* are equal.
- The measure of angle *DBC* is greater than the measure of angle *ABD*.
- Angle *DBC* is a right angle.

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Geometry and Trigonometry Sample Question 4



In the given figure, O is the center of the circle, segment BC is tangent to the circle at B, and A lies on segment OC. If the lengths of $\overline{OB}$ and $\overline{AC}$ are each 6, what is the area of the shaded region?

A) $18\sqrt{3} - 3\pi$
B) $18\sqrt{3} - 6\pi$
C) $36\sqrt{3} - 3\pi$
D) $36\sqrt{3} - 6\pi$

Since line segment BC is tangent to the circle at B, it follows that $\overline{BC} \perp \overline{OB}$, and so triangle OBC is a right triangle with its right angle at B. Since $OB = 6$ and $\overline{OB}$ and $\overline{OA}$ are both radii of the circle, $OA = 6$. Since $OC = OA + AC$, $OC = 6 + 6$, or 12. Thus, triangle OBC is a right triangle with the length of the hypotenuse ($OC = 12$) twice the length of one of its legs ($OB = 6$). It follows that triangle OBC is a 30°-60°-90° triangle with its 30° angle at C and its 60° angle at O. The area of the shaded region is the area of triangle OBC minus the area of the sector bounded by radii OA and OB.

In the 30°-60°-90° triangle OBC, the length of side OB, which is opposite the 30° angle, is 6. Thus, the length of side BC, which is opposite the 60° angle, is $6\sqrt{3}$. Hence, the area of triangle OBC is $\frac{1}{2}(6)(6\sqrt{3}) = 18\sqrt{3}$. Since the sector bounded by radii OA and OB has central angle 60°, the area of this sector is $\frac{60}{360} = \frac{1}{6}$ of the area of the circle. Since the circle has radius 6, its area is $\pi(6)^2 = 36\pi$, and so the area of the sector is $\frac{1}{6}(36\pi) = 6\pi$. Therefore, the area of the shaded region is $18\sqrt{3} - 6\pi$, which is choice B.

Note some of the key concepts that were used in sample 4:

- A tangent to a circle is perpendicular to the radius of the circle drawn to the point of tangency.
- Properties of 30°-60°-90° triangles.
- Area of a circle.
- The area of a sector with central angle $x°$ is equal to $\frac{x}{360}$ of the area of the entire circle.

STRATEGY

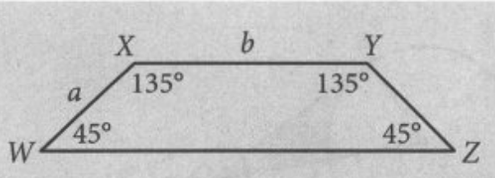
On complex multistep questions such as sample 4, start by identifying the task (finding the area of the shaded region) and considering the intermediate steps that you'll need to solve for (the area of triangle OBC and the area of sector OBA) in order to get to the final answer. Breaking up this question into a series of smaller questions will make it more manageable.

REMINDER

Arc length, area of a sector, and central angle are all proportional to each other in a circle. This proportionality is written as $\frac{\text{arc length}}{\text{circumference}} = \frac{\text{central angle}}{360 \text{ degrees}} = \frac{\text{area of a sector}}{\text{area of a circle}}$.

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Geometry and Trigonometry Sample Question 5



Trapezoid $WXYZ$ is shown. How much greater is the area of this trapezoid than the area of a parallelogram with side lengths a and b and base angles of measures 45° and 135°?

A) $\frac{1}{2}a^2$

B) $\sqrt{2}a^2$

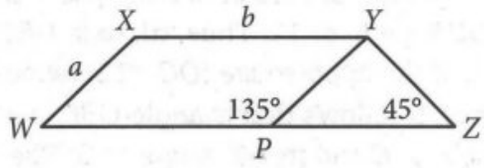
C) $\frac{1}{2}ab$

D) $\sqrt{2}ab$

STRATEGY

Note how drawing the parallelogram within trapezoid $WXYZ$ makes it much easier to compare the areas of the two shapes, minimizing the amount of calculation needed to arrive at the solution. Be on the lookout for time-saving shortcuts such as this one.

In the figure, draw a line segment from Y to the point P on side WZ of the trapezoid such that $\angle YPW$ has measure 135°, as shown in the figure below.



Since in trapezoid $WXYZ$ side XY is parallel to side WZ, it follows that $WXYP$ is a parallelogram with side lengths a and b and base angles of measure 45° and 135°. Thus, the area of the trapezoid is greater than the area of a parallelogram with side lengths a and b and base angles of measure 45° and 135° by the area of triangle PYZ. Since $\angle YPW$ has measure 135°, it follows that $\angle YPZ$ has measure 45°. Hence, triangle PYZ is a 45°-45°-90° triangle with legs of length a. Therefore, its area is $\frac{1}{2}a^2$, which is choice A.

Note some of the key concepts that were used in sample 5:

- Properties of trapezoids and parallelograms.
- Area of a 45°-45°-90° triangle.

Some questions in the Math section may ask you to find the area, surface area, or volume of an object, possibly in context.

Geometry and Trigonometry Sample Question 6

A laboratory supply company produces graduated cylinders, each with an internal radius of 2 inches and an internal height between 7.75 inches and 8 inches. What is one possible volume, rounded to the nearest cubic inch, of a graduated cylinder produced by this company?

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The volume of a cylinder can be found by using the formula $V = \pi r^2 h$, where r is the radius of the circular base and h is the height of the cylinder. The smallest possible volume, in cubic inches, of a graduated cylinder produced by the laboratory supply company can be found by substituting 2 for r and 7.75 for h, giving $V = \pi(2^2)(7.75)$. This gives a volume of approximately 97.39 cubic inches, which rounds to 97 cubic inches. The largest possible volume, in cubic inches, can be found by substituting 2 for r and 8 for h, giving $V = \pi(2^2)(8)$. This gives a volume of approximately 100.53 cubic inches, which rounds to 101 cubic inches. Therefore, the possible volumes are all the integers greater than or equal to 97 and less than or equal to 101, which are 97, 98, 99, 100, and 101. Any of these numbers may be entered as the correct answer.

Coordinate Geometry

Questions in the Geometry and Trigonometry domain may ask you to use the coordinate plane and equations of lines and circles to describe figures. You may be asked to create the equation of a circle given the figure or use the structure of a given equation to determine a property of a figure in the coordinate plane. You should know that the graph of $(x - a)^2 + (y - b)^2 = r^2$ in the xy-plane is a circle with center (a, b) and radius r.

Geometry and Trigonometry Sample Question 7

$x^2 + (y + 1)^2 = 4$

The graph of the given equation in the xy-plane is a circle. A new circle is drawn whose center is 1 unit up from this circle's center and whose radius is 1 more than this circle's radius. Which of the following is an equation of this new circle?

A) $x^2 + y^2 = 5$
B) $x^2 + y^2 = 9$
C) $x^2 + (y + 2)^2 = 5$
D) $x^2 + (y + 2)^2 = 9$

The graph of the given equation $x^2 + (y + 1)^2 = 4$ in the xy-plane is a circle with center $(0, -1)$ and radius $\sqrt{4} = 2$. If the center of the new circle is 1 unit up from the center of the given circle, the center of the new circle will be $(0, 0)$. If the radius of the new circle is 1 more than the given circle, the radius of the new circle will be 3. Therefore, an equation of the new circle in the xy-plane is $x^2 + y^2 = 3^2 = 9$, so choice B is correct.

REMINDER

You should know that the graph of $(x - a)^2 + (y - b)^2 = r^2$ in the xy-plane is a circle with center (a, b) and radius r. You should also be comfortable finding the center or radius of a circle from an equation not written in "standard form" by using the method of completing the square to rewrite the equation in standard form.

Geometry and Trigonometry Sample Question 8

$x^2 + 8x + y^2 - 6y = 24$

The graph of the given equation in the xy-plane is a circle. What is the radius of the circle?

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$x^2+2\cdot 4x+4^2+y^2-2\cdot 3y+3^2=24+4^2+3^2$

$(x+4)^2+(y-3)^2=24+16+9=49=7^2 \quad r=7$

The given equation isn't in the standard form $(x-a)^2+(y-b)^2=r^2$. You can put it in standard form by completing the square. Since the coefficient of x is 8 and the coefficient of y is −6, you can write the equation in terms of $(x+4)^2$ and $(y-3)^2$ as follows:

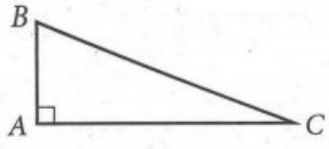
$$x^2+8x+y^2-6y=24$$
$$(x^2+8x+16)-16+(y^2-6y+9)-9=24$$
$$(x+4)^2-16+(y-3)^2-9=24$$
$$(x+4)^2+(y-3)^2=24+16+9$$
$$(x+4)^2+(y-3)^2=49$$

Since $49=7^2$, the radius of the circle is 7. (Also, the center of the circle is (−4, 3).)

Trigonometry and Radians

Questions in the Geometry and Trigonometry domain may ask you to apply the definitions from right triangle trigonometry. You should also know the definition of radian measure; you may also need to convert between angle measure in degrees and radians. You may need to evaluate trigonometric functions at benchmark angle measures such as 0, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, and $\frac{\pi}{2}$ radians (which are equal to the angle measures 0°, 30°, 45°, 60°, and 90°, respectively). You will *not* be asked for values of trigonometric functions that require a calculator.

For an acute angle, the trigonometric functions sine, cosine, and tangent can be defined using right triangles. (Note the functions are often abbreviated as sin, cos, and tan, respectively.)



For $\angle C$ in the right triangle shown:

- $\sin \angle C = \frac{AB}{BC} = \frac{\text{length of leg opposite } \angle C}{\text{length of hypotenuse}}$
- $\cos \angle C = \frac{AC}{BC} = \frac{\text{length of leg adjacent to } \angle C}{\text{length of hypotenuse}}$
- $\tan \angle C = \frac{AB}{AC} = \frac{\text{length of leg opposite } \angle C}{\text{length of leg adjacent to } \angle C} = \frac{\sin \angle C}{\cos \angle C}$

The functions will often be written as sin C, cos C, and tan C, respectively.

Note that the trigonometric functions are actually functions of the *measures* of an angle, not the angle itself. Thus, if the measure of $\angle C$ is, say, 30°, you can write sin 30°, cos 30°, and tan 30°, respectively.

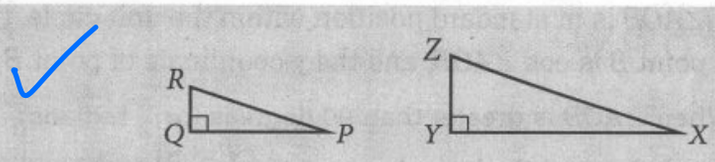
REMINDER

The acronym "SOHCAHTOA" may help you remember how to compute sine, cosine, and tangent. SOH stands for Sine equals Opposite over Hypotenuse, CAH stands for Cosine equals Adjacent over Hypotenuse, and TOA stands for Tangent equals Opposite over Adjacent.

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Also note that sine and cosine are cofunctions and that $\sin B = \frac{\text{length of leg opposite } \angle B}{\text{length of hypotenuse}} = \frac{AC}{BC} = \cos C$. This is the complementary angle relationship: $\sin x° = \cos (90° - x°)$.

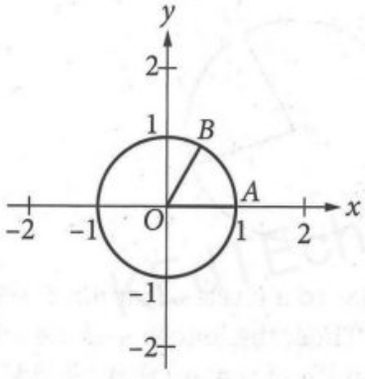
Geometry and Trigonometry Sample Question 9



In the figure, right triangle PQR is similar to right triangle XYZ, with vertices P, Q, and R corresponding to vertices X, Y, and Z, respectively. If $\cos R = 0.263$, what is the value of $\cos Z$?

By the definition of cosine, $\cos R = \frac{RQ}{RP}$ and $\cos Z = \frac{ZY}{ZX}$. Since triangle PQR is similar to triangle XYZ, with vertices P, Q, and R corresponding to vertices X, Y, and Z, respectively, the ratios $\frac{RQ}{RP}$ and $\frac{ZY}{ZX}$ are equal. Therefore, since $\cos R = \frac{RQ}{RP}$, or $\cos R = 0.263$, it follows that $\cos Z = \frac{ZY}{ZX}$, or $\cos Z = 0.263$. Note that to find the values of the trigonometric functions of $d°$, you can use *any* right triangle with an acute angle of measure $d°$ and then find the appropriate ratio of lengths of sides.

Note that since an acute angle of a right triangle has a measure between 0° and 90°, exclusive, right triangles can be used only to find values of trigonometric functions for angles with measures between 0° and 90°, exclusive. The definitions of sine, cosine, and tangent can be extended to all values. This is done using radian measure and the unit circle.



The circle above has radius 1 and is centered on the origin, O. An angle in the coordinate plane is said to be in *standard position* if it meets these two conditions: (1) its vertex lies at the origin and (2) one of its sides lies along the positive x-axis. Since $\angle AOB$, above, formed by segments OA and OB, meets both these conditions, it's in

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standard position. As segment OB, also called the *terminal side* of $\angle AOB$, rotates counterclockwise about the circle, while OA is anchored along the x-axis, the *radian* measure of $\angle AOB$ is defined to be the length s of the arc that $\angle AOB$ intercepts on the unit circle. In other words, the measure of $\angle AOB$ is s radians.

REMINDER

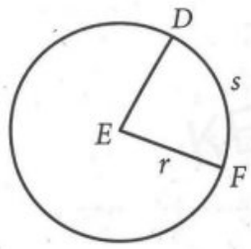
To convert from degrees to radians, multiply the number of degrees by $\frac{2\pi}{360}$ degrees. To convert from radians to degrees, multiply the number of radians by $\frac{360^\circ}{2\pi \text{ radians}}$.

When an acute $\angle AOB$ is in standard position within the unit circle, the x-coordinate of point B is $\cos \angle AOB$, and the y-coordinate of point B is $\sin \angle AOB$. When $\angle AOB$ is greater than 90 degrees (or $\frac{\pi}{2}$ radians), and point B extends beyond the boundaries of the positive x-axis and positive y-axis, the values of $\cos \angle AOB$ and $\sin \angle AOB$ may be negative depending on the coordinates of point B. For any $\angle AOB$, place $\angle AOB$ in standard position within the circle of radius 1 centered at the origin, with side OA along the positive x-axis and terminal side OB intersecting the circle at point B. Then the cosine of $\angle AOB$ is the x-coordinate of B, and the sine of $\angle AOB$ is the y-coordinate of B. The tangent of $\angle AOB$ is the sine of $\angle AOB$ divided by the cosine of $\angle AOB$.

An angle with a full rotation about point O has measure 360°. This angle intercepts the full circumference of the circle, which has length 2π. Thus, $\frac{\text{measure of an angle in radians}}{\text{measure of an angle in degrees}} = \frac{2\pi}{360^\circ}$. It follows that the measure of an angle in radians is $\frac{2\pi}{360^\circ}$ × the measure of the angle in degrees and that the measure of an angle in degrees is $\frac{360^\circ}{2\pi}$ × the measure of the angle in radians.

Also note that since a rotation of 2π about point O brings you back to the same point on the unit circle, $\sin(s + 2\pi) = \sin s$, $\cos(s + 2\pi) = \cos s$, and $\tan(s + 2\pi)$, for any radian measure s.

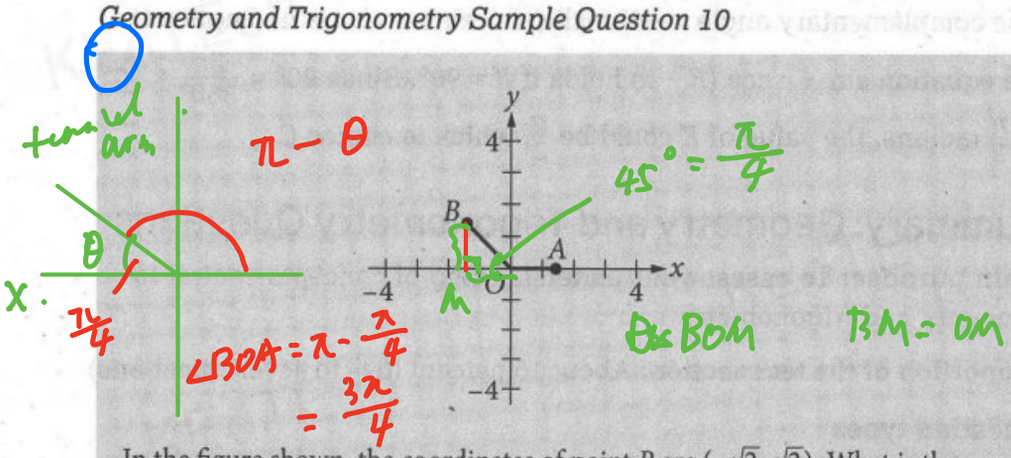
Let angle DEF be a central angle in a circle of radius r, as shown in the following figure.



A circle of radius r is similar to a circle of radius 1, with constant of proportionality equal to r. Thus, the length s of the arc intercepted by angle DEF is r times the length of the arc that would be intercepted by an angle of the same measure in a circle of radius 1. Therefore, in the figure above, $s = r \times$ (radian measure of angle DEF), or radian measure of angle $DEF = \frac{s}{r}$.

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Geometry and Trigonometry Sample Question 10



In the figure shown, the coordinates of point B are $(-\sqrt{2}, \sqrt{2})$. What is the measure, in radians, of angle AOB?

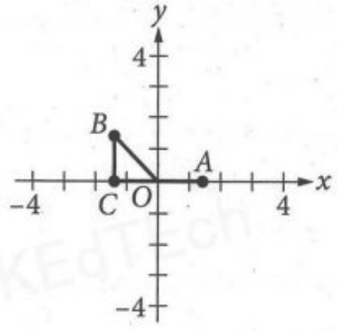
A) $\frac{\pi}{4}$

B) $\frac{\pi}{2}$

C) $\frac{3\pi}{4}$

D) $\frac{5\pi}{4}$

Let C be the point $(-\sqrt{2}, 0)$. Then triangle BOC, shown in the figure below, is a right triangle with both legs of length $\sqrt{2}$.



Hence, triangle BOC is a 45°-45°-90° triangle. Thus, angle COB has measure 45°, and angle AOB has measure $180° - 45° = 135°$. Therefore, the measure of angle AOB in radians is $135° \times \frac{2\pi}{360°} = \frac{3\pi}{4}$, which is choice C.

STRATEGY

Always be on the lookout for special right triangles. Here, noticing that segment OB is the hypotenuse of a 45°-45°-90° triangle makes this question easier to solve.

Geometry and Trigonometry Sample Question 11

$$\sin x = \cos (K - x)$$

In the given equation, the angle measures are in radians and K is a constant. Which of the following could be the value of K?

A) 0

B) $\frac{\pi}{4}$

C) $\frac{\pi}{2}$

D) π

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The complementary angle relationship for sine and cosine implies that the equation $\sin x = \cos(K - x)$ holds if $K = 90°$. Since $90° = \frac{2\pi}{360°} \times 90°$, or $\frac{\pi}{2}$ radians, the value of K could be $\frac{\pi}{2}$, which is choice C.

Summary: Geometry and Trigonometry Questions

Main purpose: To assess your understanding of concepts central to geometry and trigonometry

Proportion of the test section: About 15 percent (five to seven questions)

Question types

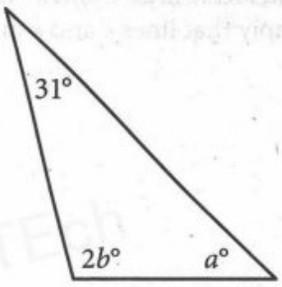
- Area and volume
- Lines, angles, and triangles
- Right triangles and trigonometry
- Circles

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CHAPTER 28

Math: Questions—Geometry and Trigonometry Drills

1



In the triangle shown, $a = 45$. What is the value of b?

A) 52
B) 59
C) 76
D) 104

2

A cube has a surface area of 54 square meters. What is the volume, in cubic meters, of the cube?

$6x^2$

A) 18
B) 27
C) 36
D) 81



3

In the xy-plane, a circle with radius 5 has center $(-8, 6)$. Which of the following is an equation of the circle?

A) $(x - 8)^2 + (y + 6)^2 = 25$
B) $(x + 8)^2 + (y - 6)^2 = 25$
C) $(x - 8)^2 + (y + 6)^2 = 5$
D) $(x + 8)^2 + (y - 6)^2 = 5$

$(x-h)^2 + (y-k)^2 = r^2$

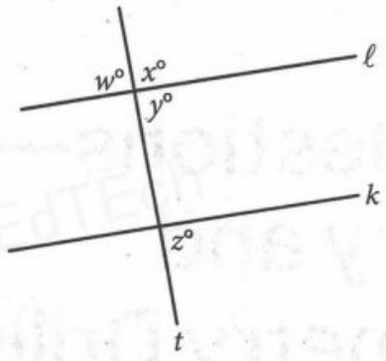
$(-8, 6)$

$(x-(-8))^2 + (y-6)^2 = r^2$

$(x+8)^2$

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4



Note: Figure not drawn to scale.

In the figure shown, line t intersects lines ℓ and k. Which of the following statements, if true, would imply that lines ℓ and k are parallel?

A) $w = y$

B) $w = z$

C) $x = z$

D) $x + y = 180$

5

In a right triangle, the tangent of one of the two acute angles is $\frac{\sqrt{3}}{3}$. What is the tangent of the other acute angle?

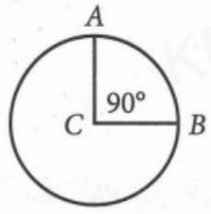
A) $-\frac{\sqrt{3}}{3}$

B) $-\frac{3}{\sqrt{3}}$

C) $\frac{\sqrt{3}}{3}$

D) $\frac{3}{\sqrt{3}}$

6



Point C is the center of the circle shown. What is the measure of angle ACB, in radians?

A) 2π

B) π

C) $\frac{\pi}{2}$

D) $\frac{\pi}{4}$

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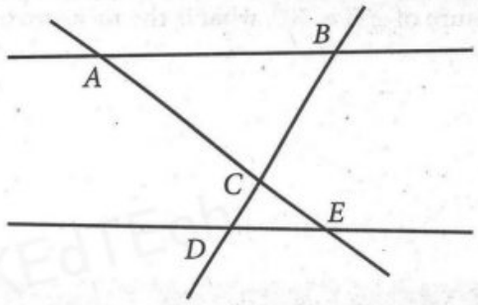
7

The length of one side of square M is 5 times the length of one side of square N. The area of square N is 361 square centimeters. What is the area, in square centimeters, of square M?

8

Triangle KLM is similar to triangle QRS, where angle K corresponds to angle Q and where angles L and R are right angles. If $\sin K = \frac{105}{233}$ and $\sin M = \frac{208}{233}$, what is the value of $\tan S$?

9



Note: Figure not drawn to scale.

In the figure shown, $\triangle ABC$ is similar to $\triangle EDC$, with $\angle BAC$ corresponding to $\angle CED$ and $\angle ABC$ corresponding to $\angle CDE$. Which of the following must be true?

A) $\overline{AE} \parallel \overline{BD}$

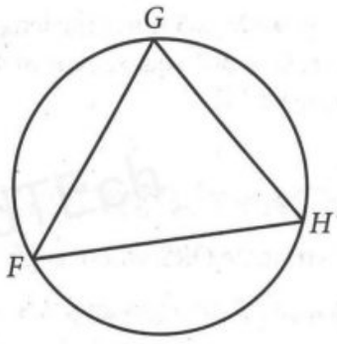
B) $\overline{AE} \perp \overline{BD}$

C) $\overline{AB} \parallel \overline{DE}$

D) $\overline{AB} \perp \overline{DE}$

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10

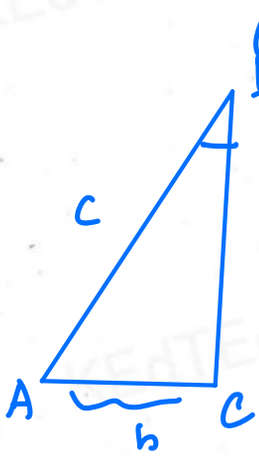


Note: Figure not drawn to scale.

Triangle *FGH* is inscribed in the circle shown. If arc *FG* is congruent to arc *GH* and the measure of $\angle G$ is 30°, what is the measure of $\angle H$?

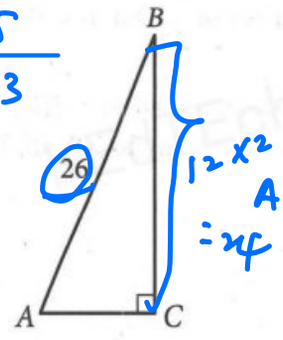
A) 30°
B) 60°
C) 75°
D) 120°

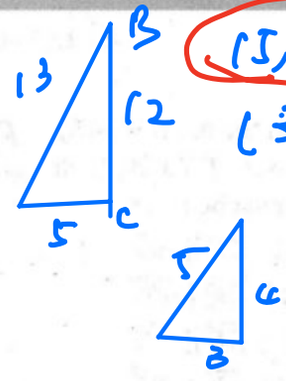
11



$\sin B = \frac{b}{c} = \frac{5}{13}$

$\frac{5}{13} = \frac{x}{26}$





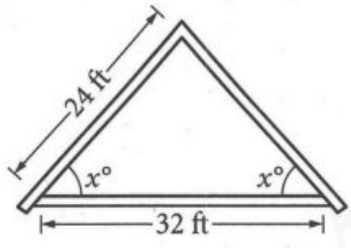
(5, 12, 13)
(3, 4, 5)

Triangle *ABC* shown is a right triangle, and $\sin B = \frac{5}{13}$. What is the length of side $\overline{BC}$?

24

12

An architect drew the sketch shown while designing a house roof. The dimensions shown are for the interior of the triangle.



Note: Figure not drawn to scale.

What is the value of $\cos x$?

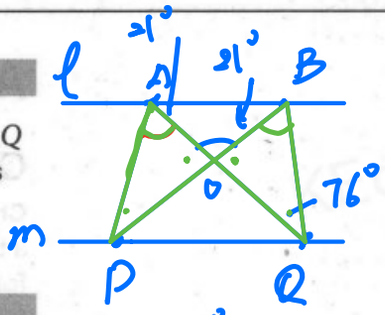
$\frac{2}{5}$

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13

Line ℓ is parallel to line m. Points A and B lie on line ℓ, and points P and Q lie on line m. If $\angle ABP$ and $\angle QAB$ each have measure 21° and $\angle AQB$ has measure 76°, what is the measure, in degrees, of $\angle PAQ$?

62°



$\angle AOB = 138°$

$\angle AOB = \angle ABP + \angle PBQ$

$138° = 76° + \angle PBQ$

$\angle PBQ = 138° - 76°$

$= 62°$

14



The semicircle shown has a radius of r inches, and chord $\overline{CD}$ is parallel to the diameter $\overline{AB}$. If the length of $\overline{CD}$ is $\frac{2}{3}$ of the length of $\overline{AB}$, what is the distance between the chord and the diameter in terms of r?

A) $\frac{1}{3}\pi r$

B) $\frac{2}{3}\pi r$

C) $\frac{\sqrt{2}}{2}r$

D) $\frac{\sqrt{5}}{3}r$

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CHAPTER 25

Math: Questions—Problem-Solving and Data Analysis

Problem-Solving and Data Analysis questions in the Math section assess your ability to use your math skills and knowledge to solve problems, many of which are set in context. Some of the questions in this content domain may ask you to create a representation of a problem, consider the units involved, pay attention to the meaning of quantities, know and use different properties of mathematical operations and representations, and apply key principles of statistics and probability. Special focus in this domain is given to mathematical models. Models are representations of real-life contexts. They help us to explain or interpret the behavior of certain components of a system and to predict results that are as yet unobserved or unmeasured. You may be asked to create and use a model and to understand the distinction between the predictions of a model and the data that have been collected.

Some of the questions in this content domain may involve quantitative reasoning about ratios, rates, percentages, and proportional relationships and may require understanding and applying unit rates. Some problems may be set in contexts, which may address topics in science, social science, and real-world settings.

Some questions may present information about the relationship between two variables in a graph, scatterplot, table, or another form and ask you to analyze and draw conclusions about the given information. These questions assess your understanding of the key properties of, and the differences between, linear, quadratic, and exponential relationships and how these properties apply to the corresponding contexts.

The Problem-Solving and Data Analysis domain may also include questions that assess your understanding of essential concepts in statistics and probability. You may be asked to analyze univariate data (data involving one variable) presented in dot plots, histograms, box plots, bar graphs, and frequency tables, or bivariate data (data involving two variables) presented in scatterplots, line graphs, and two-way tables. This includes computing, comparing, and interpreting measures of center; interpreting measures of spread; describing overall patterns; and recognizing the effects of outliers on measures of center and spread.

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These questions may test your understanding of the conceptual meaning of standard deviation (although you won't be asked to calculate a standard deviation).

Other questions in this domain may ask you to estimate the probability of an event, employing different approaches, rules, or probability models. Special attention is given to the notion of conditional probability, which is tested using two-way tables and in other ways.

Some questions may present you with a description of a study and ask you to decide what conclusion is most appropriate based on the design of the study. Some questions may ask about using data from a sample to draw conclusions about an entire population. These questions may also assess conceptual understanding of the margin of error (although you won't be asked to calculate a margin of error) when a population mean or proportion is estimated from sample data. Other questions may ask about making conclusions about cause-and-effect relationships between two variables.

Problem-Solving and Data Analysis questions include both multiple-choice questions and student-produced response questions.

Let's explore the content, skills, and knowledge assessed by Problem-Solving and Data Analysis questions in the Math section.

Ratio, Proportion, Units, and Percentages

Ratio and proportion is one of the major ideas in math. Introduced well before high school, ratio and proportion is a theme throughout mathematics and has many applications in daily life, career work, and higher-level math courses.

Problem-Solving and Data Analysis Sample Question 1

On Thursday, 240 adults and children attended a show. The ratio of adults to children was 5 to 1. How many children attended the show?

A) 40
B) 48
C) 192
D) 200

REMINDER

A ratio represents a relationship between quantities, not the actual quantities themselves. Fractions are an especially effective way to represent and work with ratios.

Because the ratio of adults to children was 5 to 1, there were 5 adults for every 1 child. Thus, of every 6 people who attended the show, 5 were adults and 1 was a child. In fractions, $\frac{5}{6}$ of the 240 who attended were adults and $\frac{1}{6}$ were children. Therefore, $\frac{1}{6} \times 240$, or 40 children attended the show, which is choice A.

Ratios in the Math section may be expressed in the forms 3 to 1, 3:1, $\frac{3}{1}$, or simply 3.

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Problem-Solving and Data Analysis Sample Question 2

On an architect's drawing of the floor plan for a house, 1 inch represents 3 feet. If a room is represented on the floor plan by a rectangle that has sides of lengths 3.5 inches and 5 inches, what is the actual floor area of the room, in square feet?

A) 17.5
B) 51.0
C) 52.5
D) 157.5

Because 1 inch represents 3 feet, the actual dimensions of the room are 3 × 3.5, or 10.5 feet, and 3 × 5, or 15 feet. Therefore, the floor area of the room is 10.5 × 15, or 157.5 square feet, which is choice D.

Another classic example of a ratio is the comparison of the length of an object to the length of its shadow. At a given location and time of day, it might be true that a fence post that has a height of 4 feet casts a shadow that is 6 feet long. This ratio of the height of the object to the length of its shadow, 4 to 6, or 2 to 3, remains the same for any object at the same location and time. This could be considered a unit rate: the ratio of the height of the object to the length of its shadow would be equivalent to 1 to $\frac{2}{3}$ or the unit rate $\frac{2}{3}$-foot change in height of the object for every 1-foot change in length of shadow. So, for example, a tree that casts a shadow that's 18 feet long has a height of $18 \times \frac{2}{3}$, or 12 feet. In this situation, in which one variable quantity is always a fixed constant times another variable quantity, the two quantities are said to be directly proportional.

Thus, if variables x and y are said to be directly proportional, then there exists some number k such that $y = kx$, where k is a nonzero constant. The constant k is called the constant of proportionality.

In the preceding example, you'd say that the height of an object is directly proportional to the length of the object's shadow, with constant of proportionality $\frac{2}{3}$. So if you let L be the length of the shadow and H be the height of the object, then $H = \frac{2}{3}L$.

Notice that both L and H are lengths, so the constant of proportionality, $\frac{H}{L} = \frac{2}{3}$, has no units. In contrast, let's consider sample question 2 again. On the scale drawing, 1 inch represents 3 feet. The length of an actual measurement is directly proportional to its length on the scale drawing. But to find the constant of proportionality, you need to keep track of units: $\frac{3 \text{ feet}}{1 \text{ inch}} = \frac{36 \text{ inches}}{1 \text{ inch}} = 36$. Hence, if S is a length on the scale drawing that corresponds to an actual length of R, then $R = 36S$, where R and S have the same units.

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Many questions in the Math section require you to pay attention to units. Some questions in the Problem-Solving and Data Analysis content domain require you to convert units either between the U.S. customary system and the metric system or within those systems.

Problem-Solving and Data Analysis Sample Question 3

Scientists estimate that the Pacific Plate, one of Earth's tectonic plates, has moved about 1,060 kilometers in the past 10.3 million years. About how far, in miles, has the Pacific Plate moved during this same time period? (Use 1 mile = 1.6 kilometers.)

A) 165
B) 398
C) 663
D) 1,696

STRATEGY

Pay close attention to units, and convert units if required by the question. Writing out the unit conversion as a series of multiplication steps, as seen here, will help ensure accuracy. Intermediate units should cancel (as do the kilometers in sample 3), leaving you with the desired unit.

Because 1 mile = 1.6 kilometers, the distance is

$$1{,}060 \text{ kilometers} \times \frac{1 \text{ mile}}{1.6 \text{ kilometers}} = 662.5 \text{ miles}$$

, which is about 663 miles. Therefore, the correct answer is choice C.

Problem-Solving and Data Analysis Sample Question 4

County Y consists of two districts. One district has an area of 30 square miles and a population density of 370 people per square mile, and the other district has an area of 50 square miles and a population density of 290 people per square mile. What is the population density, in people per square mile, for all of County Y?

320 people

Note that this example is in the student-produced response (SPR) format and therefore has no answer choices. You have to generate and enter the response yourself.

The first district has an area of 30 square miles and a population density of 370 people per square mile, so its total population is

$$30 \text{ square miles} \times \frac{370 \text{ people}}{\text{square mile}}$$

, or 11,100 people. The other district has an area of 50 square miles and a population density of 290 people per square mile, so its total population is 50 square miles $\times \frac{290 \text{ people}}{\text{square mile}}$, or 14,500 people. Thus, County Y has a total population of 11,100 + 14,500 = 25,600 people and a total area of 30 + 50 = 80 square miles. Therefore, the population density of County Y is $\frac{25{,}600}{80}$, or 320 people per square mile.

The Problem-Solving and Data Analysis domain also includes questions involving percentages, which are a type of proportion. These questions may involve the concepts of percentage increase and percentage decrease.

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Problem-Solving and Data Analysis Sample Question 5

A furniture store buys its furniture from a wholesaler. For a particular style of table, the usual price of the table is 75% more than the cost of the table from the wholesaler. During a sale, the store sells the table for 15% more than the cost from the wholesaler. If the sale price of the table is $299, what is the usual price for the table?

A) $359
B) $455
C) $479
D) $524

The sale price of the table was $299. This is equal to the cost from the wholesaler plus 15%. Thus, $299 = 1.15 (cost from the wholesaler), and the cost from the wholesaler is $\frac{\$299}{1.15} = \$260$. The usual price is the cost from the wholesaler, $260, plus 75%. Therefore, the usual price the store charges for the table is $1.75 \times \$260 = \$455$, which is choice B.

Interpreting Relationships Presented in Scatterplots, Graphs, Tables, and Equations

The behavior of a variable and the relationship between two variables in a context may be explored by considering data presented in scatterplots, tables, and graphs.

The relationship between two quantitative variables may be modeled by a function or an equation. The model may allow very accurate predictions, as, for example, models used in physical sciences, or may only describe a general trend, with considerable variability between the actual and predicted values, as, for example, models used in behavioral and social sciences.

Questions in the Math section may assess your ability to understand and analyze relationships between two variables, the properties of the functions used to model these relationships, and the conditions under which a model is considered to be an appropriate representation of the data. Problem-Solving and Data Analysis questions of these sorts focus on linear, quadratic, and exponential relationships.

REMINDER

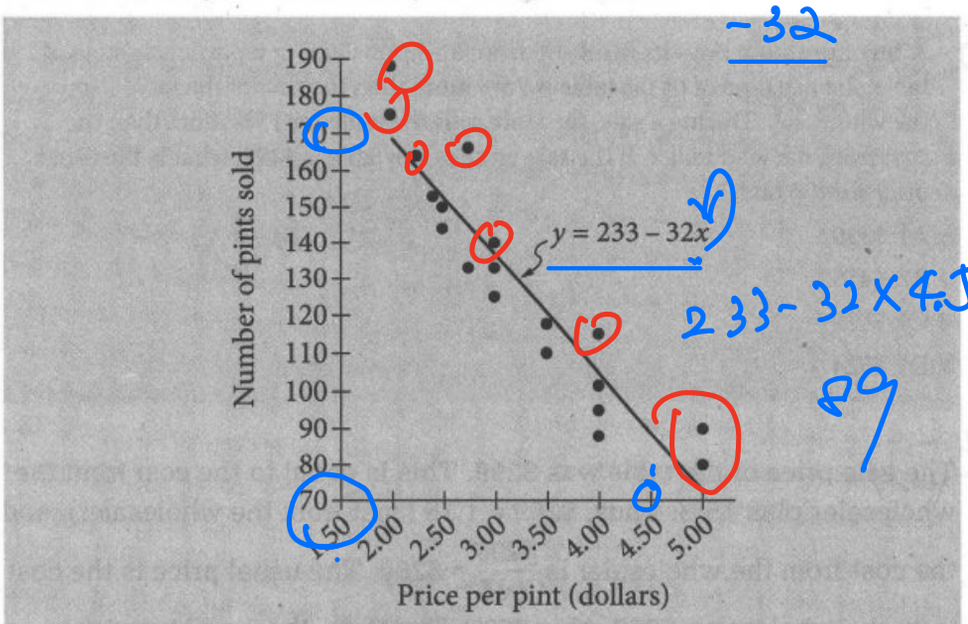
Percent is a type of proportion that means "per 100"; 20%, for instance, means 20 out of (or per) 100. Percent increase or decrease is calculated by finding the difference between two quantities, then dividing the difference by the original quantity and multiplying the result by 100.

REMINDER

The abilities of interpreting and synthesizing data from scatterplots, graphs, and tables are widely applicable in college and in many careers and thus are tested in the Math section.

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Problem-Solving and Data Analysis Sample Question 6



A grocery store sells pints of raspberries and sets the price per pint each week. The scatterplot above shows the price and the number of pints of raspberries sold for 19 weeks, along with a line of best fit for the data and an equation for the line of best fit.

Several different questions could be asked about this context.

A. According to the line of best fit, how many pints of raspberries would the grocery store be predicted to sell in a week when the price of raspberries is $4.50 per pint?

Because the line of best fit has equation $y = 233 - 32x$, where x is the price, in dollars, for a pint of raspberries and y is the predicted number of pints of raspberries sold, the number of pints the store would be predicted to sell in a week when the price of raspberries is $4.50 per pint is $233 - 32(4.50) = 89$ pints.

B. For how many of the 19 weeks shown was the number of pints of raspberries sold greater than the number predicted by the line of best fit?

For a given week, the number of pints of raspberries sold is greater than the number predicted by the line of best fit if and only if the point representing that week lies above the line of best fit. For example, at the price of $5 per pint, the number sold in two different weeks was approximately 80 and 90, which is more than the 73 predicted by the line of best fit. Of the 19 points, 8 lie above the line of best fit, so there were 8 weeks in which the number of pints sold was greater than what was predicted by the line of best fit.

C. What is the best interpretation of the slope of the line of best fit in this context?

REMINDER

A line of best fit is a straight line that best represents the association shown on a scatterplot. It's often written in $y = a + bx$ form.

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In the Math section, this question would be followed by multiple-choice answer options. The slope of the line of best fit is −32. This means that the correct answer would state that for each dollar that the price of a pint of raspberries increases, the store is predicted to sell 32 fewer pints of raspberries.

D. What is the best interpretation of the y-intercept of the line of best fit in this context?

In the Math section, this question would be followed by multiple-choice answer options.

In this context, the *y*-intercept doesn't represent a likely scenario, so it can't be accurately interpreted in this context. According to the model, the *y*-intercept means that if the store sold raspberries for $0 per pint—that is, if the store gave raspberries away—233 people would be expected to accept the free raspberries. However, it's not realistic that the store would give away raspberries, and if it did, it's likely that far more people would accept the free raspberries. Also notice that in this case, the leftmost line on the graph is not the *y*-axis. The lower-left corner shows the *x*- and *y*-coordinates of (1.5, 70), not (0, 0).

The fact that the *y*-intercept indicates that 233 people would accept free raspberries is one limitation of the model. Another limitation is that for a price of $7.50 per pint or above, the model predicts that a negative number of people would buy raspberries, which is impossible. In general, you should be cautious about applying a model for values outside of the given data. In this example, you should only be confident in the prediction of sales for prices between $2 and $5 because those are the lowest and highest prices for which data are shown.

Giving a line of best fit, as in this example, assumes that the relationship between the variables is best modeled by a linear function, but that isn't always true. In the Math section, you may see data that are best modeled by a linear, quadratic, or exponential model.

Problem-Solving and Data Analysis Sample Question 7

Time (hours)	Number of bacteria
0	1,000
1	4,000
2	16,000
3	64,000

The table gives the initial number (at time $t = 0$) of bacteria placed in a growth medium and the number of bacteria in the growth medium each hour for 3 hours. Which of the following functions best models the number of bacteria, $N(t)$, after t hours?

A) $N(t) = 4{,}000t$

B) $N(t) = 1{,}000 + 3{,}000t$

C) $N(t) = 1{,}000(4^{-t})$

D) $N(t) = 1{,}000(4^{t})$

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STRATEGY

To determine whether a relationship is linear or exponential, examine the change in the quantity between successive time periods. If the difference in the quantity is constant, the relationship is linear. If the ratio in the quantity is constant (for instance, 4 times greater than the preceding time period), the relationship is exponential.

The given choices are linear and exponential models. If a quantity is increasing linearly with time, then the *difference* in the quantity between successive time periods is constant. If a quantity is increasing exponentially with time, then the *ratio* in the quantity between successive time periods is constant. According to the table, after each hour, the number of bacteria in the culture is 4 times as great as it was the preceding hour: $\frac{4,000}{1,000} = \frac{16,000}{4,000} = \frac{64,000}{16,000} = 4$. That is, for each increase of 1 in t, the value of $N(t)$ is multiplied by 4. At $t = 0$, which corresponds to the time when the culture was placed in the medium, there were 1,000 bacteria. This is modeled by the exponential function $N(t) = 1,000(4^t)$, which has the value 1,000 at $t = 0$ and increases by a factor of 4 for each increase of 1 in the value of t. Choice D is the correct answer.

The Math section may include questions that require you to know the difference between linear and exponential growth.

Problem-Solving and Data Analysis Sample Question 8

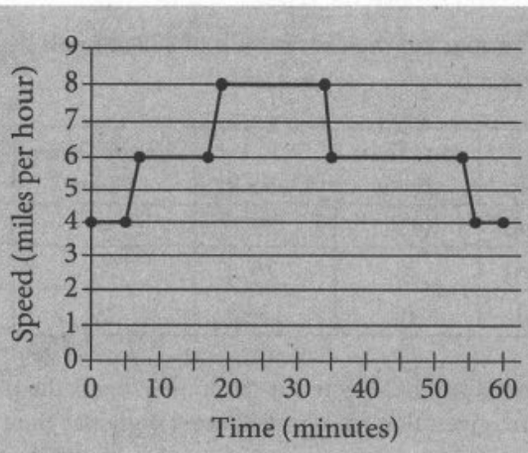
Every month, Jamal adds 2 new books to his library. Which of the following types of functions best models the number of books in Jamal's library as a function of time?

A) Increasing linear
B) Decreasing linear
C) Increasing exponential
D) Decreasing exponential

Over equal intervals, linear functions increase or decrease by a constant amount, while exponential functions increase or decrease by a constant factor. Since the number of books is increasing by a constant amount (2 books) over equal intervals (each month), the function is linear. Also, since the number of books is increasing as time increases, the function is increasing, and therefore choice A is correct.

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Problem-Solving and Data Analysis Sample Question 9



One evening, Maria walks, jogs, and runs for a total of 60 minutes. The graph above shows Maria's speed during the 60 minutes. Which segment of the graph represents the times when Maria's speed is the greatest?

A) The segment from (17, 6) to (19, 8)
B) The segment from (19, 8) to (34, 8)
C) The segment from (34, 8) to (35, 6)
D) The segment from (35, 6) to (54, 6)

The correct answer is choice B. Because the vertical coordinate represents Maria's speed, the part of the graph with the greatest vertical coordinate represents the times when Maria's speed is the greatest. This is the highest part of the graph, the segment from (19, 8) to (34, 8), when Maria runs at 8 miles per hour (mph). Choice A represents the time during which Maria's speed is increasing from 6 to 8 mph; choice C represents the time during which Maria's speed is decreasing from 8 to 6 mph; and choice D represents the longest period of Maria moving at the same speed (6 mph), not the times when Maria's speed is the greatest.

More Data and Statistics

Some questions in the Problem-Solving and Data Analysis domain may assess your ability to understand and analyze data presented in a table, bar graph, histogram, dot plot, box plot, line graph, or other display.

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Problem-Solving and Data Analysis Sample Question 10

The given table summarizes the distribution of 200 animals by type of animal and mass, in kilograms.

	Less than 45 kg	45–55 kg	Greater than 55 kg	Total
Cheetah	12	59	4	75
Leopard	24	76	25	125
Total	36	135	29	200

One of these animals will be selected at random. What is the probability of selecting a leopard, given that the animal's mass is greater than 55 kilograms?

A) $\frac{25}{29}$

B) $\frac{1}{5}$

C) $\frac{29}{200}$

D) $\frac{1}{8}$

$\frac{25}{29}$

The probability of selecting a leopard given that the animal's mass is greater than 55 kg is the number of leopards with a mass greater than 55 kg divided by the total number of animals with a mass greater than 55 kg. According to the table, there are 29 animals with a mass greater than 55 kg. Of these 29 animals, 25 are leopards. Therefore, if the animal has a mass greater than 55 kg, the probability that the animal is a leopard is $\frac{25}{29}$. The correct answer is choice A.

Sample 10 is an example of a conditional probability, the probability of an event given that another event is known to have occurred. The question asks for the probability that an animal chosen at random is a leopard given that the animal has a mass greater than 55 kg.

REMINDER

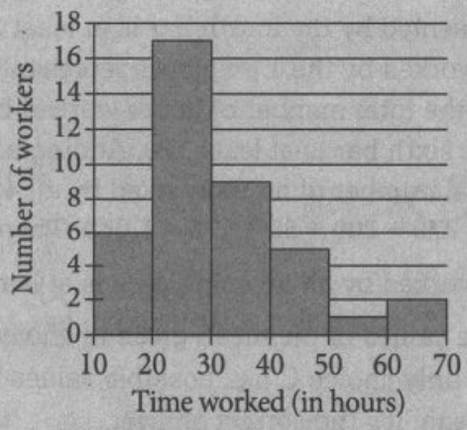
Mean and median are measures of center for a data set, while range and standard deviation are measures of spread.

You may be asked to answer questions that involve a measure of center for a data set: the mean or the median. A question may ask you to draw conclusions about one or more of these measures of center even if the exact values cannot be calculated. To recall briefly:

- The *mean* of a set of numerical values is the sum of all the values divided by the number of values in the set.
- The *median* of a set of numerical values is the middle value when the values are listed in increasing (or decreasing) order. If the set has an even number of values, then the median is the average of the two middle values. While technically, the median could be any number between the two middle values, it is most often computed as the average of the two middle values and will always be computed this way for questions in the Math section.

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Problem-Solving and Data Analysis Sample Question 11



The histogram shown summarizes the distribution of time worked last week, in hours, by the 40 employees of a landscaping company. In the histogram, the first bar represents all workers who worked at least 10 hours but less than 20 hours; the second bar represents all workers who worked at least 20 hours but less than 30 hours; and so on. Which of the following could be the median and mean amount of time worked, in hours, for the 40 employees?

A) Median = 22, mean = 23
B) Median = 24, mean = 22
C) Median = 26, mean = 32
D) Median = 32, mean = 30

Note: In the Math section, all questions that include a histogram will include a description of the boundary condition if interpretation of the boundary is required. That is, the question will include a description of how to interpret the values represented by a bar and whether to include the left endpoint but not the right endpoint or the other way around.

If the number of hours the 40 employees worked is listed in increasing order, the median will be a value between the 20th and the 21st numbers in the list. The first 6 numbers in the list will be workers represented by the first bar; hence, each of the first 6 numbers will be at least 10 but less than 20. The next 17 numbers—that is, the 7th through the 23rd numbers in the list—will be workers represented by the second bar; hence, each of the next 17 numbers will be at least 20 but less than 30. Thus, the 20th and the 21st numbers in the list will be at least 20 but less than 30. Therefore, any of the median values in choices A, B, or C is possible, but the median value in choice D isn't.

Now let's find the possible values of the mean. Each of the 6 employees represented by the first bar worked at least 10 hours but less than 20 hours. Thus, the total number of hours worked by these 6 employees is at least 60. Similarly, the total number of hours worked by the 17 employees represented by the second bar is at least 340; the total

REMINDER
The distribution of a variable provides all possible values of the variable and how often they occur.

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number of hours worked by the 9 employees represented by the third bar is at least 270; the total number of hours worked by the 5 employees represented by the fourth bar is at least 200; the total number of hours worked by the 1 employee represented by the fifth bar is at least 50; and the total number of hours worked by the 2 employees represented by the sixth bar is at least 120. Adding all these hours shows that the total number of hours worked by all 40 employees is at least 60 + 340 + 270 + 200 + 50 + 120 = 1,040. Therefore, the mean number of hours worked by all 40 employees is at least $\frac{1,040}{40} = 26$. Therefore, only the values of the mean given in choices C and D are possible. Because only choice C has possible values for both the median and the mean, it's the correct answer.

A data set may have a few values that are much larger or smaller than the rest of the values in the set. These values are called *outliers*. An outlier may represent an important piece of data. For example, if a data set consists of rates of a certain illness in various cities, a data point with a very high value could indicate a serious health issue to be investigated.

In general, outliers affect the mean more than the median. Therefore, outliers that are larger than the rest of the points in the data set tend to make the mean greater than the median, and outliers that are smaller than the rest of the points in the data set tend to make the mean less than the median. In sample 11, the mean was larger than the median due to the unusually large amount of time worked by a few employees.

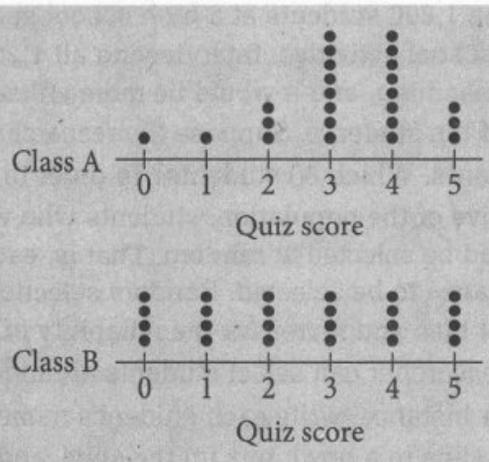
REMINDER

You won't be asked to calculate the standard deviation of a set of data in the Math section, but you will be expected to demonstrate an understanding of what standard deviation measures.

The mean and the median are different ways to describe the center of a data set. Another key characteristic of a data set is the amount of variability, or spread, in the data. One measure of spread is the range, which is equal to the maximum value minus the minimum value. Another measure of spread is the standard deviation, which is a measure of how dispersed, or far away, the points in the data set are from the mean value. A low standard deviation indicates the points in the data set are clustered around the mean value, and a high standard deviation indicates the points in the data set are more spread out from the mean value. In the Math section, you won't be asked to compute the standard deviation of a data set, but you do need to understand that a larger standard deviation corresponds to a data set whose values are more spread out from the mean value.

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Problem-Solving and Data Analysis Sample Question 12



The dot plots show the distributions of scores on a current events quiz for two classes of 24 students each. Which of the following statements about the standard deviations of the two distributions is true?

A) The standard deviation of quiz scores in Class A is less than that of quiz scores in Class B.

B) The standard deviation of quiz scores in Class A is greater than that of quiz scores in Class B.

C) The standard deviation of quiz scores in Class A is equal to that of quiz scores in Class B.

D) There is not enough information to compare the standard deviations.

In Class A, the mean score is between 3 and 4. The large majority of scores are 3 and 4, with only a few scores of 0, 1, 2, and 5. In Class B, the mean score is 2.5, and scores are evenly distributed across all possible scores, with many scores not close to the mean score. Because the scores in Class A are more closely clustered around the mean, the standard deviation of the scores in Class A is smaller. The correct answer is choice A.

A *population parameter* is a numerical value that describes a characteristic of a population. For example, the percentage of registered voters who would vote for a certain candidate is a parameter describing the population of registered voters in an election. In another example, the average income of a household in a city is a parameter describing the population of households in that city. We often don't know the value of the population parameter; thus, an essential purpose of statistics is to estimate a population parameter based on a sample from the population. A common example is election polling, where researchers will interview a random sample of registered voters to estimate the proportion of all registered voters who plan to vote for a certain candidate. The precision of the estimate depends on the variability of the sample data and the sample size. For instance, if household incomes in a city vary widely or the sample is small, the estimate that comes from a sample may differ considerably from the actual value for the population parameter.

STRATEGY

When asked to compare the standard deviations of two data sets, first locate the mean approximately. Then ask yourself which data set has values that are more closely clustered around the mean. That data set will have the smaller standard deviation.

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For example, a researcher wants to estimate the mean number of hours each week that the 1,200 students at a high school spend on campus engaged in after-school activities. Interviewing all 1,200 students would be time consuming, and it would be more efficient to survey a random sample of the students. Suppose the researcher has time to interview 80 students. Which 80 students? In order to have a sample that's representative of the population, students who will participate in the study should be selected at random. That is, each student must have the same chance to be selected. Random selection is essential in protecting against bias and increases the reliability of any estimates calculated. The researcher can select students at random in several different ways; for instance, write each student's name on a slip of paper, put all the slips in a bowl, mix up the slips, and then draw 80 names from the bowl. In practice, a computer is often used to select participants at random.

If you don't select a random sample, the sampling method used may introduce bias. For example, if you found 80 students from those attending a game of the school's football team, those people would be more likely to be interested in sports and, in turn, an interest in sports might be related to the average amount of time the students spend on the internet. The result would be that the average time those 80 students spend on the internet might not be an accurate estimate of the average amount of time all students at the school spend on the internet.

Suppose you select 80 students at random from the 1,200 students at the high school. You ask them how much time they spend on campus engaged in after-school activities each week, and you calculate that the mean time is 7.9 hours per week. You also find that 6 of the 80 students spend less than 1 hour each week on campus engaged in after-school activities. Based on these results, what conclusions should be made about the entire population of 1,200 students?

Because the sample was selected at random, the mean of 7.9 hours is a plausible estimate of the mean time spent each week on campus engaged in after-school activities by all 1,200 students. Also, we can use the sample data to estimate how many students spend less than 1 hour on campus engaged in after-school activities each week. In the sample, the percentage is $\frac{6}{80}$, or 7.5%. Applying this percentage to the entire population of 1,200 students, the best estimate is that 90 students at the school spend less than 1 hour per week on campus engaged in after-school activities.

REMINDER

You won't need to calculate margins of error in the Math section, but you should understand what the concept means and be able to interpret margins of error in context.

However, the estimates of the population parameters need to be interpreted carefully. An essential part of statistics is accounting for the variability of the estimate. The estimates above are reasonable, but they're unlikely to be exactly correct. Statistical analysis can also describe how far from the estimates the actual values are expected at most to be. To describe the precision of an estimate, statisticians use the concept of *margin of error*. You won't be expected to compute a

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margin of error in the Math section, but you should understand how sample size affects margin of error and how to interpret a given margin of error in context.

If the example above had been part of a Math section question, you might have been given survey results indicating that for a random sample of 80 students, the estimated mean was 7.9 hours with an associated margin of error of 0.8 hours. An appropriate interpretation of these results is that it's plausible that the mean number of hours for all 1,200 students in the population is greater than 7.9 − 0.8, or 7.1 hours, but less than 7.9 + 0.8, or 8.7 hours. There are two key points to note.

1. The value of the margin of error is affected by two factors: the variability in the data and the sample size. The larger the standard deviation, the larger the margin of error; the smaller the standard deviation, the smaller the margin of error. Furthermore, increasing the size of the random sample provides more information and typically reduces the margin of error.
2. The margin of error applies to the estimated value of the population parameter only; it doesn't inform the estimated value for an individual. In the example, plausible values for the population mean are in the interval from 7.1 hours to 8.7 hours. The time, in hours, that an individual spends on the internet may or may not fall in this interval.

Problem-Solving and Data Analysis Sample Question 13

A quality control researcher at an electronics company is testing the life of the company's batteries in a certain camera. The researcher selects 100 batteries at random from the daily output of the batteries and finds that the life of the batteries has a mean of 342 pictures with an associated margin of error of 18 pictures. Which of the following is the most appropriate conclusion based on these data?

A) All the batteries produced by the company that day have a life between 324 and 360 pictures.

B) All the batteries ever produced by the company have a life between 324 and 360 pictures.

C) It is plausible that the mean life of batteries produced by the company that day is between 324 and 360 pictures.

D) It is plausible that the mean life of all the batteries ever produced by the company is between 324 and 360 pictures.

The correct answer is choice C. Choices A and B are incorrect because the margin of error gives information about the mean life of all batteries produced by the company that day, not about the life of any individual battery. Choice D is incorrect because the sample of batteries was taken from the population of all the batteries produced by the company on that day. The population of all batteries the company ever produced may have a different mean life because of changes in the formulation of the batteries, wear on machinery, improvements in production processes, and many other factors.

STRATEGY

When a margin of error is provided, determine the value to which the margin of error applies. If the margin of error is associated with a mean, then the margin of error describes the plausible values for individual values in the population.

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The statistics examples discussed so far are largely based on investigations intended to estimate some characteristic of a group: the mean amount of time students spend on the internet, the mean life of a battery, and the percentage of registered voters who plan to vote for a candidate. Another primary focus of statistics is to investigate relationships between variables and to draw conclusions about cause and effect. For example, does a new type of physical therapy help people recover from knee surgery faster? For such a study, some people who have had knee surgery will be randomly assigned to the new therapy or to the usual therapy. The medical results of these patients can be compared. The key questions from a statistical viewpoint are

- Is it appropriate to generalize from the sample of patients in the study to the entire population of people who are recovering from knee surgery?
- Is it appropriate to conclude that the new therapy caused any difference in the results for the two groups of patients?

REMINDER

In order for the results of a study to be generalized to an entire population, random sampling is needed. In order for a cause-and-effect relationship to be established, random assignment of individuals to treatments is needed.

The answers depend on the use of random selection and random assignment.

- If the subjects in the sample of a study were selected at random from the entire population in question, the results can be generalized to the entire population because random sampling ensures that each individual has the same chance to be selected for the sample.
- If the subjects in the sample were randomly assigned to treatments, it may be appropriate to make conclusions about cause and effect because the treatment groups will be roughly equivalent at the beginning of the experiment other than the treatment they receive.

This can be summarized in the following table.

	Subjects selected at random	Subjects not selected at random
Subjects randomly assigned to treatments	• Results can be generalized to the entire population. • Conclusions about cause and effect can appropriately be drawn.	• Results *cannot* be generalized to the entire population. • Conclusions about cause and effect can appropriately be drawn.
Subjects not randomly assigned to treatments	• Results can be generalized to the entire population. • Conclusions about cause and effect should *not* be drawn.	• Results *cannot* be generalized to the entire population. • Conclusions about cause and effect should *not* be drawn.

The previous example discussed treatments in a medical experiment. The word *treatment* refers to any factor that is deliberately varied in an experiment.

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Problem-Solving and Data Analysis Sample Question 14

A community center offers a Spanish course. This year, all students in the course were offered additional audio lessons they could take at home. The students who took these additional audio lessons did better in the course than students who didn't take the additional audio lessons. Based on these results, which of the following is the most appropriate conclusion?

A) Taking additional audio lessons will cause an improvement for any student who takes any foreign language course.

B) Taking additional audio lessons will cause an improvement for any student who takes a Spanish course.

C) Taking additional audio lessons was the cause of the improvement for the students at the community center who took the Spanish course.

D) No conclusion about cause and effect can be made regarding students at the community center who took the additional audio lessons at home and their performance in the Spanish course.

The correct answer is choice D. The better results of these students may have been a result of being more motivated, as shown in their willingness to do extra work, and not the additional audio lessons. Choice A is incorrect because no conclusion about cause and effect is possible without random assignment to treatments and because the sample was only students taking a Spanish course, so no conclusion can be appropriately made about students taking all foreign language courses. Choice B is incorrect because no conclusion about cause and effect is possible without random assignment to treatments and because the students taking a Spanish course at the community center is not a random sample of all students who take a Spanish course. Choice C is incorrect because the students taking the Spanish course at the community center weren't randomly assigned to use the additional audio lessons or to not use the additional audio lessons.

STRATEGY

Be wary of conclusions that claim a cause-and-effect relationship or that generalize a conclusion to a broader population. Before accepting a conclusion, assess whether the subjects were selected at random from the broader population and whether subjects were randomly assigned to treatments.

Summary: Problem-Solving and Data Analysis Questions

Main purpose: To assess your understanding of proportional relationships, percentages, and probability as well as your ability to use data to analyze and solve problems

Proportion of the test section: About 15 percent (five to seven questions)

Question types

- Ratios, rates, proportional relationships, and units
- Percentages
- One-variable data: distributions and measures of center and spread
- Two-variable data: models and scatterplots
- Probability and conditional probability
- Inference from sample statistics and margin of error
- Evaluating statistical claims: observational studies and experiments

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CHAPTER 26

Math: Questions—Problem-Solving and Data Analysis Drills

1

Makayla is planning an event in a 5,400-square-foot room. If there should be at least 8 square feet per person, what is the maximum number of people that could attend this event?

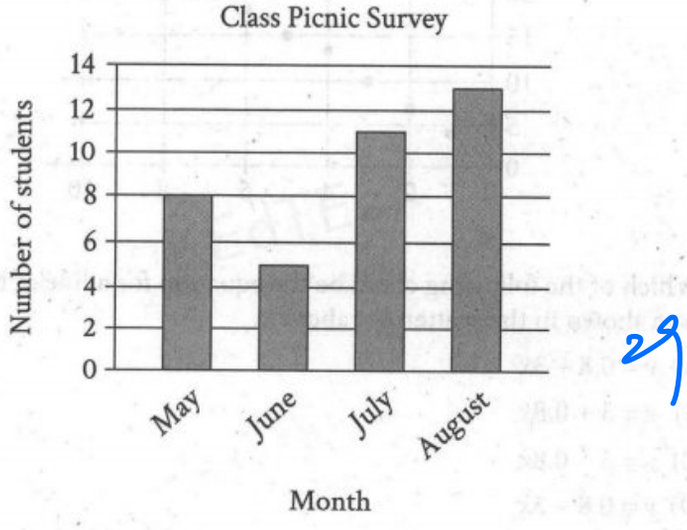
A) 588
B) 675
C) 15,274
D) 43,200

2

Water flows from a pipe at a rate of 6.0 gallons per minute. How many gallons of water will flow from the pipe in 8.4 minutes?

3

The bar graph shows the results from a survey in which a group of students was asked during which month they prefer to have the class picnic.



Based on the graph, how many students did not respond "May"?

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4

A fish hatchery has three tanks for holding fish before they are introduced into the wild. Ten fish weighing less than 5 ounces are placed in tank A. Eleven fish weighing at least 5 ounces but no more than 13 ounces are placed in tank B. Twelve fish weighing more than 13 ounces are placed in tank C. Which of the following could be the median of the weights, in ounces, of these 33 fish?

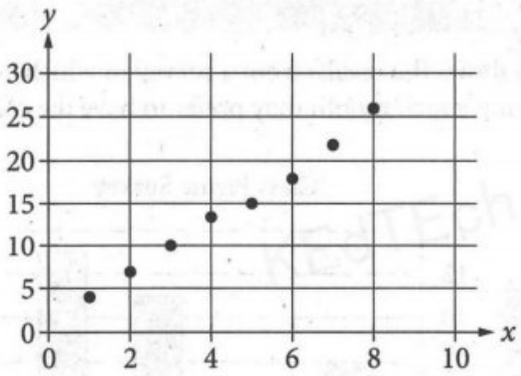
A) 4.5
B) 8
C) 13.5
D) 15

5

A data set of 27 different numbers has a mean of 33 and a median of 33. A new data set is created by adding 7 to each number in the original data set that is greater than the median and subtracting 7 from each number in the original data set that is less than the median. Which of the following measures does NOT have the same value in both the original and new data sets?

A) Median
B) Mean
C) Sum of the numbers
D) Standard deviation

6



Which of the following could be the equation for a line of best fit for the data shown in the scatterplot above?

A) $y = 0.8 + 3x$
B) $y = 3 + 0.8x$
C) $y = 3 - 0.8x$
D) $y = 0.8 - 3x$

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