CHAPTER 23

Math: Questions—Advanced Math

Advanced Math questions in the Math section address topics that are especially important for students to master before studying higher-level math concepts. Chief among these topics is the understanding of the structure of expressions and the ability to analyze, manipulate, and rewrite these expressions. Questions in this content domain also include ones involving reasoning with more complex equations and interpreting and building functions.

Algebra questions focus on the mastery of linear equations and inequalities, systems of linear equations, and linear functions. By contrast, Advanced Math questions focus on the ability to work with and analyze more complex equations. Advanced Math questions may require you to demonstrate procedural skill in adding, subtracting, and multiplying polynomials and in factoring polynomials. You may also be required to work with expressions involving integer and rational exponents, radicals, or fractions with a variable in the denominator. Questions in this domain may ask you to solve quadratic, radical, rational, polynomial, or absolute value equations. They may also ask you to solve a system consisting of a linear equation and a nonlinear equation. You may also be required to manipulate an equation in several variables to isolate a quantity of interest.

Some questions in Advanced Math may ask you to build a quadratic or an exponential function or an equation that describes a context or to interpret the function, the graph of the function, or the solution to an equation in terms of the context.

Advanced Math questions may assess your ability to recognize structure. Expressions and equations that appear complex may use repeated terms or repeated expressions. By noticing these patterns, the complexity of a problem can be reduced. Structure may be used to factor or otherwise rewrite an expression, to solve a quadratic or other equation, or to draw conclusions about the context represented by an expression, equation, or function. You may be asked to identify or derive the form of an expression, equation, or function that reveals information about the expression, equation, or function or the context it represents.

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Advanced Math questions may also assess your understanding of functions and their graphs. A question may require you to demonstrate your understanding of function notation, including interpreting an expression in which the argument of a function is an expression rather than a variable. The questions may assess your understanding of how the algebraic properties of a function relate to the geometric characteristics of its graph.

Advanced Math questions include both multiple-choice questions and student-produced response questions. Although you may always use a calculator in the Math section, you must decide whether using it is an effective strategy for a given question.

Let's consider the content, skills, and knowledge assessed by Advanced Math questions.

Operations with Polynomials and Rewriting Expressions

Questions in the Math section may assess your ability to add, subtract, and multiply polynomials.

Advanced Math Sample Question 1

$$x^3 + 6x^2 + 7x - 6 = (x^2 + bx - 2)(x + 3)$$

In the given equation, b is a constant. If the equation is true for all values of x, what is the value of b?

A) 2
B) 3
C) 7
D) 9

The given equation shows two polynomials that are equivalent. To find the value of b, use the distributive property to rewrite the polynomial on the right-hand side of the given equation and then combine like terms so that the polynomial on the right-hand side is in the same form as the polynomial on the left-hand side.

$$\begin{aligned} x^3 + 6x^2 + 7x - 6 &= (x^2 + bx - 2)(x + 3) \\ &= (x^3 + bx^2 - 2x) + (3x^2 + 3bx - 6) \\ &= x^3 + (3 + b)x^2 + (3b - 2)x - 6 \end{aligned}$$

Since the two polynomials are equal for all values of x, the coefficient of matching powers of x should be the same. Therefore, comparing the coefficients of $x^3 + 6x^2 + 7x - 6$ and $x^3 + (3 + b)x^2 + (3b - 2)x - 6$ reveals that $3 + b = 6$ and $3b - 2 = 7$. Solving either of these equations gives $b = 3$, which is choice B.

STRATEGY

The skills and knowledge tested in Advanced Math questions build on the knowledge and skills tested by Algebra questions. Develop proficiency with the tested Algebra skills and knowledge before tackling Advanced Math questions.

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Questions may also ask you to use structure to rewrite expressions. For example, the expression may be in the form of a common pattern, such as a difference of squares, where using the difference-of-squares structure allows for efficient rewriting of the expression as the product of two binomials, as shown in sample 2.

Advanced Math Sample Question 2

Which of the following is equivalent to $16s^4 - 4t^2$?

A) $4(s^2 - t)(4s^2 + t)$

B) $4(4s^2 - t)(s^2 + t)$

C) $4(2s^2 - t)(2s^2 + t)$

D) $(8s^2 - 2t)(8s^2 + 2t)$

A close examination reveals that the given expression follows the difference of two perfect squares pattern, $x^2 - y^2$, which factors as the product $(x - y)(x + y)$. The expression $16s^4 - 4t^2$ is also the difference of two squares: $16s^4 - 4t^2 = (4s^2)^2 - (2t)^2$. Therefore, it can be factored as $(4s^2)^2 - (2t)^2 = (4s^2 - 2t)(4s^2 + 2t)$. This expression can be rewritten as $(4s^2 - 2t)(4s^2 + 2t) = 2(2s^2 - t)(2)(2s^2 + t)$, or $4(2s^2 - t)(2s^2 + t)$, which is choice C.

Alternatively, a 4 could be factored out of the given expression: $4(4s^4 - t^2)$. The expression inside the parentheses is a difference of two squares. Therefore, it can be further factored as $4(2s^2 - t)(2s^2 + t)$.

STRATEGY

Advanced Math questions require a high comfort level working with quadratic equations and expressions, including multiplying polynomials and factoring. Recognizing classic quadratic patterns such as $x^2 - y^2 = (x - y)(x + y)$ can also improve your speed and accuracy.

Advanced Math Sample Question 3

Which expression is equivalent to $xy^2 + 2xy^2 + 3xy$?

A) $2xy^2 + 3xy$

B) $3xy^2 + 3xy$

C) $6xy^4$

D) $6xy^5$

There are three terms in the expression, the first two of which are like terms. The like terms can be added together by adding their coefficients: $xy^2 + 2xy^2 + 3xy = (xy^2 + 2xy^2) + 3xy$, which is equivalent to $3xy^2 + 3xy$. Therefore, choice B is correct.

Quadratic Functions and Equations

Advanced Math questions may require you to build a quadratic function or an equation to represent a context.

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Advanced Math Sample Question 4

A car is traveling at x feet per second. The driver sees a red light ahead, and after 1.5 seconds reaction time, the driver applies the brake. After the brake is applied, the car takes $\frac{x}{24}$ seconds to stop, during which time the average speed of the car is $\frac{x}{2}$ feet per second. If the car travels 165 feet from the time the driver saw the red light to the time it comes to a complete stop, which of the following equations can be used to find the value of x?

A) $x^2 + 48x - 3{,}960 = 0$
B) $x^2 + 48x - 7{,}920 = 0$
C) $x^2 + 72x - 3{,}960 = 0$
D) $x^2 + 72x - 7{,}920 = 0$

STRATEGY

Sample 4 requires careful translation of an in-context problem into an algebraic equation. It pays to be deliberate and methodical when translating such problems in the Math section into equations.

During the 1.5-second reaction time, the car is still traveling at x feet per second, so it travels a total of $1.5x$ feet. The average speed of the car during the $\frac{x}{24}$-second braking interval is $\frac{x}{2}$ feet per second, so over this interval the car travels $\left(\frac{x}{2}\right)\left(\frac{x}{24}\right) = \frac{x^2}{48}$ feet. Since the total distance the car travels from the time the driver saw the red light to the time the car comes to a complete stop is 165 feet, you have the equation $\frac{x^2}{48} + 1.5x = 165$. This quadratic equation can be rewritten in standard form by subtracting 165 from each side and then multiplying each side by 48, giving $x^2 + 72x - 7{,}920 = 0$, which is choice D.

STRATEGY

Questions in the Advanced Math domain may ask you to solve a quadratic equation. Practice using the various methods (below) until you're comfortable with all of them.

1. Factoring
2. Completing the square
3. Using the quadratic formula
4. Graphing using technology
5. Using structure

Be prepared to identify which strategy may be the most effective for solving a given quadratic equation.

Some questions in the Advanced Math domain may ask you to solve a quadratic equation. You must determine an appropriate procedure: factoring, completing the square, using the quadratic formula, graphing using technology, or using structure. It may be more efficient to solve a quadratic equation if you can make use of one of the following facts:

- The sum of the distinct solutions of $x^2 + bx + c = 0$ is $-b$.
- The product of the distinct solutions of $x^2 + bx + c = 0$ is c.

Each of these facts can be observed from examining the factored form of a quadratic. If r and s are distinct solutions of $x^2 + bx + c = 0$, then $x^2 + bx + c = (x - r)(x - s)$. Thus, $b = -(r + s)$ and $c = (-r)(-s) = rs$. **Note:** To use either of these facts, the coefficient of x^2 must be equal to 1.

Advanced Math Sample Question 5

What are the solutions to the equation $x^2 - 3 = x$?

A) $\frac{-1 \pm \sqrt{11}}{2}$
B) $\frac{-1 \pm \sqrt{13}}{2}$
C) $\frac{1 \pm \sqrt{11}}{2}$
D) $\frac{1 \pm \sqrt{13}}{2}$

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The equation can be solved by using the quadratic formula or by completing the square. Let's use the quadratic formula. First, subtract x from each side of $x^2 - 3 = x$ to put the equation into standard form: $x^2 - x - 3 = 0$. The quadratic formula states that the solutions, x, of the equation $ax^2 + bx + c = 0$ are $\frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$. For the equation $x^2 - x - 3 = 0$, $a = 1$, $b = -1$, and $c = -3$. Substituting these values into the quadratic formula gives $x = \frac{-(-1) \pm \sqrt{(-1)^2 - 4(1)(-3)}}{2(1)}$. This equation can be rewritten as $x = \frac{1 \pm \sqrt{1-(-12)}}{2}$, or $x = \frac{1 \pm \sqrt{13}}{2}$, which is choice D.

REMINDER

The quadratic formula states that the solutions x of the equation $ax^2 + bx + c = 0$ are $x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$.

Advanced Math Sample Question 6

If $x > 0$ and $2x^2 + 3x - 2 = 0$, what is the value of x?

Note that this example is in the student-produced response (SPR) format and therefore has no answer choices. You have to generate and enter the response yourself.

The left-hand side of the given equation can be factored: $2x^2 + 3x - 2 = (2x - 1)(x + 2)$. Therefore, the given equation can be rewritten as $(2x - 1)(x + 2) = 0$, which implies that either $2x - 1 = 0$, which gives $x = \frac{1}{2}$, or $x + 2 = 0$, which gives $x = -2$. Since $x > 0$, the value of x is $\frac{1}{2}$.

STRATEGY

Pay close attention to all the details in each question. In sample 6, x can equal $\frac{1}{2}$ or -2, but since the question states that $x > 0$, the value of x must be $\frac{1}{2}$.

Advanced Math Sample Question 7

What is the sum of the solutions to $(2x - 1)^2 = (x + 2)^2$?

If a and b are real numbers and $a^2 = b^2$, then either $a = b$ or $a = -b$. It follows then that since $(2x - 1)^2 = (x + 2)^2$, either $2x - 1 = x + 2$ or $2x - 1 = -(x + 2)$. In the first case, subtracting x and adding 1 to both sides of the equation yields $x = 3$. In the second case, applying the distributive property on the right side of the equation gives $2x - 1 = -x - 2$.

Adding x and 1 to both sides of this equation yields $3x = -1$. It follows that $x = -\frac{1}{3}$. Therefore, the sum of the solutions of $(2x - 1)^2 = (x + 2)^2$ is $3 + \left(-\frac{1}{3}\right)$, or $\frac{8}{3}$.

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Exponential Functions, Equations, and Expressions and Radicals

Some Advanced Math questions may ask you to build a function that models a given context. Exponential functions model situations in which a quantity is multiplied by a constant factor for each time period. An exponential function can be increasing with time, in which case it models exponential growth, or it can be decreasing with time, in which case it models exponential decay.

Advanced Math Sample Question 8

A researcher estimates that the population of a city is increasing at an annual rate of 0.6%. If the current population of the city is 80,000, which of the following expressions appropriately models the population of the city t years from now according to the researcher's estimate?

A) $80{,}000(1 + 0.006)^t$

B) $80{,}000(1 + 0.006^t)$

C) $80{,}000 + 1.006^t$

D) $80{,}000(0.006^t)$

REMINDER

A quantity that grows or decays by a fixed percent at regular intervals is said to exhibit exponential growth or decay, respectively.

Exponential growth is represented by the function $y = a(1 + r)^t$, while exponential decay is represented by the function $y = a(1 - r)^t$, where y is the new population, a is the initial population, r is the rate of growth or decay, and t is the number of time intervals that have elapsed.

According to the researcher's estimate, the population is increasing by 0.6% each year. Since 0.6% is equal to 0.006, after the first year the population is $80{,}000 + 0.006(80{,}000) = 80{,}000(1 + 0.006)$. After the second year, the population is $80{,}000(1 + 0.006) + 0.006(80{,}000)(1 + 0.006) = 80{,}000(1 + 0.006)^2$. Similarly, after t years, the population will be $80{,}000(1 + 0.006)^t$ according to the researcher's estimate. This is choice A.

A well-known example of exponential decay is the decay of a radioactive isotope. One example involves iodine-131, a radioactive isotope used in some medical treatments. The half-life of iodine-131 is 8.02 days; that is, after 8.02 days, half of the iodine-131 in a sample will have decayed. Suppose a sample with a mass of A milligrams of iodine-131 decays for d days. Every 8.02 days, the quantity of iodine-131 is multiplied by $\frac{1}{2}$, or 2^{-1}. In d days, a total of $\frac{d}{8.02}$ different 8.02-day periods will have passed, and so the original quantity will have been multiplied by 2^{-1} a total of $\frac{d}{8.02}$ times. Therefore, the mass, in milligrams, of iodine-131 remaining in the sample will be $A(2^{-1})^{\frac{d}{8.02}} = A\left(2^{-\frac{d}{8.02}}\right)$.

In the preceding discussion, we used the identity $\frac{1}{2} = 2^{-1}$. Advanced Math questions may require you to apply this and other laws of exponents as well as the relationship between powers and radicals.

Some Advanced Math questions may ask you to use properties of exponents to rewrite expressions.

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Advanced Math Sample Question 9

Which of the following is equivalent to $\left(\frac{1}{\sqrt{x}}\right)^n$, where $x > 0$?

A) $x^{\frac{n}{2}}$

B) $x^{-\frac{n}{2}}$

C) $x^{n+\frac{1}{2}}$

D) $x^{n-\frac{1}{2}}$

The expression $\sqrt{x}$ is equal to $x^{\frac{1}{2}}$. Thus, $\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$, and $\left(\frac{1}{\sqrt{x}}\right)^n = \left(x^{-\frac{1}{2}}\right)^n = x^{-\frac{n}{2}}$. Choice B is the correct answer.

An Advanced Math question may also ask you to solve a radical equation. In solving radical equations, you may square both sides of an equation. Since squaring both sides of an equation may result in an extraneous solution, you may end up with a root to the simplified equation that isn't a root to the original equation. Thus, when solving a radical equation, you must check any solution you get in the original equation.

STRATEGY

Practice your exponent rules. Know, for instance, that $\sqrt{x} = x^{\frac{1}{2}}$ and that $\frac{1}{\sqrt{x}} = x^{-\frac{1}{2}}$.

Advanced Math Sample Question 10

$$x - 12 = \sqrt{x + 44}$$

What are all possible solutions to the given equation?

A) 5

B) 20

C) −5 and 20

D) 5 and 20

Squaring each side of $x - 12 = \sqrt{x + 44}$ gives

$$(x - 12)^2 = (\sqrt{x + 44})^2$$

$$(x - 12)^2 = x + 44$$

$$x^2 - 24x + 144 = x + 44$$

$$x^2 - 25x + 100 = 0$$

$$(x - 5)(x - 20) = 0$$

The solutions to this quadratic equation are $x = 5$ and $x = 20$. However, since the first step was to square each side of the given equation, which may have introduced an extraneous solution, you need to check $x = 5$ and $x = 20$ in the original equation. Substituting 5 for x gives

$$5 - 12 = \sqrt{5 + 44}$$

$$-7 = \sqrt{49}$$

This isn't a true statement (since $\sqrt{49}$ represents the principal square root, or only the positive square root, 7), so $x = 5$ is *not* a solution to $x - 12 = \sqrt{x + 44}$. Substituting 20 for x gives

$$20 - 12 = \sqrt{20 + 44}$$

$$8 = \sqrt{64}$$

STRATEGY

A good strategy to use when solving radical equations is to square both sides of the equation. When doing so, however, be sure to check the solutions in the original equation, as you may end up with a root that isn't a solution to the original equation.

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This is a true statement, so $x = 20$ is a solution to $x - 12 = \sqrt{x + 44}$. Therefore, the only solution to the given equation is 20, which is choice B.

Solving Rational Equations

Questions in the Advanced Math domain may assess your ability to work with rational expressions, including fractions with a variable in the denominator. This may include finding the solution to a rational equation.

Advanced Math Sample Question 11

$$\frac{3}{t+1} = \frac{2}{t+3} + \frac{1}{4}$$

If t is a solution to the given equation and $t > 0$, what is the value of t?

STRATEGY

When solving for a variable in an equation involving fractions, a good first step is to clear the variable out of the denominators of the fractions. Remember that you can only multiply both sides of an equation by an expression when you know the expression can't be equal to 0.

Since $t > 0$, both sides of the given equation can be multiplied by the lowest common denominator, which is $4(t + 1)(t + 3)$. The resulting equivalent equation won't have any fractions, and the variable will no longer be in the denominator. This gives $12(t + 3) = 8(t + 1) + (t + 1)(t + 3)$. This equation can be rewritten as $12t + 36 = (8t + 8) + (t^2 + 4t + 3)$, or $12t + 36 = t^2 + 12t + 11$. Subtracting $12t$ and 36 from both sides of this equation yields $0 = t^2 - 25$. This equation can be rewritten by factoring the right-hand side as $0 = (t - 5)(t + 5)$. By the zero product property, $t - 5 = 0$ or $t + 5 = 0$. Therefore, the solutions to the equation are $t = 5$ or $t = -5$. Since $t > 0$, the value of t is 5.

Systems of Equations

Questions in the Advanced Math domain may ask you to solve a system of equations in two variables in which one equation is linear and the other equation is quadratic or another nonlinear equation.

Advanced Math Sample Question 12

$$3x + y = -3$$
$$(x + 1)^2 - 4(x + 1) - 6 = y$$

If (x, y) is a solution to the given system of equations and $y > 0$, what is the value of y?

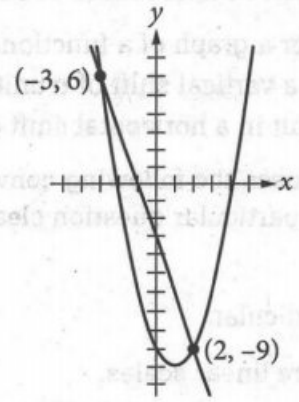
STRATEGY

The first step used to solve this example was substitution, an approach you may use on questions about finding the solution to a system of linear equations as well. The second step used to solve this example was taken after noticing that $(x + 1)$ can be treated as a variable.

One method for solving systems of equations is substitution. If the first equation is solved for y, it can be substituted in the second equation. Subtracting $3x$ from each side of the first equation gives you $y = -3 - 3x$, which can be rewritten as $y = -3(x + 1)$. Substituting $-3(x + 1)$ for y in the second equation gives you $(x + 1)^2 - 4(x + 1) - 6 = -3(x + 1)$. Since the factor $(x + 1)$ appears as a squared term and a linear term, the equation can be thought of as a quadratic equation in the variable $(x + 1)$, so collecting the terms and setting the expression equal to 0 gives you $(x + 1)^2 - (x + 1) - 6 = 0$. Rewriting this equation by factoring gives you $((x + 1) - 3)((x + 1) + 2) = 0$, or $(x - 2)(x + 3) = 0$.

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Thus, either $x = 2$, which gives $y = -3 - 3(2) = -9$; or $x = -3$, which gives $y = -3 - 3(-3) = 6$. Therefore, the solutions to the system are $(2, -9)$ and $(-3, 6)$. Since the question states that $y > 0$, the value of y is 6.



The solutions of the system are given by the intersection points of the two graphs. Questions in the SAT Math section may assess this or other relationships between algebraic and graphical representations of functions.

Relationships Between Algebraic and Graphical Representations of Functions

A function f has a graph in the xy-plane, which is the graph of the equation $y = f(x)$, or, equivalently, consists of all ordered pairs $(x, f(x))$. Some Advanced Math questions may assess your ability to relate properties of the function f to properties of its graph and vice versa. You may be required to apply some of the following relationships:

- **Intercepts.** The x-intercepts of the graph of f correspond to values of x such that $f(x) = 0$, which corresponds to values of x where the graph intersects with the x-axis, also known as the function's zeros; if the function f has no zeros, its graph has no x-intercepts and vice versa. The y-intercept of the graph of f corresponds to the value of $f(0)$, or the value of y where the graph intersects with the y-axis. If $x = 0$ is not in the domain of f, the graph of f has no y-intercept and vice versa.
- **Domain and range.** The domain of f is the set of all x for which $f(x)$ is defined. The range of f is the set of all y such that $y = f(x)$ for some value of x in the domain. The domain and range can be found from the graph of f as the set of all x-coordinates and y-coordinates, respectively, of points on the graph.
- **Maximum and minimum values.** The maximum and minimum values of f can be found by locating the highest and the lowest points on the graph, respectively. For example, suppose P is the highest point on the graph of f. Then the y-coordinate of P is the maximum value of f, and the x-coordinate of P is where f takes on its maximum value.

REMINDER

The domain of a function is the set of all values for which the function is defined. The range of a function is the set of all values that correspond to the values in the domain, given the relationship defined by the function, or the set of all outputs that are associated with all the possible inputs.

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- **Increasing and decreasing.** The graph of f shows the intervals over which the function f is increasing and decreasing.
- **End behavior.** The graph of f can indicate if $f(x)$ increases or decreases without limit as x increases or decreases without limit.
- **Transformations.** For a graph of a function f, a change of the form $f(x) + a$ will result in a vertical shift of a units, and a change of the form $f(x - a)$ will result in a horizontal shift of a units.

STRATEGY

Don't assume the size of the units on the two axes are equal unless the question states they're equal or you can conclude they're equal from the information given.

Note: The Math section uses the following conventions about graphs in the xy-plane unless a particular question clearly states or shows a different convention:

- The axes are perpendicular.
- Scales on the axes are linear scales.
- The size of the units on the two axes can't be assumed to be equal unless the question states they're equal or you're given enough information to conclude they're equal.
- The values on the horizontal axis increase as you move to the right.
- The values on the vertical axis increase as you move up.

Advanced Math Sample Question 13

The graph of which of the following functions in the xy-plane has x-intercepts at -4 and 5?

A) $f(x) = (x + 4)(x - 5)$
B) $g(x) = (x - 4)(x + 5)$
C) $h(x) = (x - 4)^2 + 5$
D) $k(x) = (x + 5)^2 - 4$

The x-intercepts of the graph of a function correspond to the zeros of the function. All the functions in the choices are defined by quadratic equations, so the answer must be a quadratic function. If a quadratic function has x-intercepts at -4 and 5, then the values of the function at -4 and 5 are each 0; that is, the zeros of the function occur at $x = -4$ and at $x = 5$. Since the function is defined by a quadratic equation and has zeros at $x = -4$ and $x = 5$, it must have $(x + 4)$ and $(x - 5)$ as factors. Therefore, choice A, $f(x) = (x + 4)(x - 5)$, is correct.

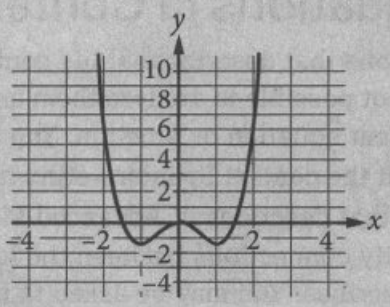
STRATEGY

Another way to think of sample 13 is to ask yourself, "Which answer choice represents a function that has values of 0 when $x = -4$ and $x = 5$?"

The graph in the xy-plane of each of the functions in the previous example is a parabola. Using the defining equations, you can tell that the graph of g has x-intercepts at 4 and -5; the graph of h has its vertex at $(4, 5)$; and the graph of k has its vertex at $(-5, -4)$.

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Advanced Math Sample Question 14



The function $f(x) = x^4 - 2.4x^2$ is graphed in the xy-plane where $y = f(x)$, as shown. If k is a constant such that the equation $f(x) = k$ has 4 solutions, which of the following could be the value of k?

A) 1
B) 0
C) −1
D) −2

Choice C is correct. Since $f(x) = x^4 - 2.4x^2$, the equation $f(x) = k$, or $x^4 - 2.4x^2 = k$, will have four solutions if and only if the graph of the horizontal line with equation $y = k$ intersects the graph of f at four points. The graph shows that of the given choices, only for choice C, −1, does the graph of $y = -1$ intersect the graph of f at four points.

Function Notation

The Math section may assess your understanding of function notation. In such cases, you must be able to evaluate a function given the rule that defines it, and if the function describes a context, you may need to interpret the value of the function in the context. A question may ask you to interpret a function when an expression, such as $2x$ or $x + 1$, is used as the argument instead of the variable x.

Advanced Math Sample Question 15

If $g(x) = 2x + 1$ and $f(x) = g(x) + 4$, what is $f(2)$?

You're given $f(x) = g(x) + 4$; therefore, $f(2) = g(2) + 4$. To determine the value of $g(2)$, use the function $g(x) = 2x + 1$. Thus, $g(2) = 2(2) + 1$, and therefore, $g(2) = 5$. Substituting $g(2)$ gives $f(2) = 5 + 4$, or $f(2) = 9$.

Alternatively, since $f(x) = g(x) + 4$ and $g(x) = 2x + 1$, it follows that $f(x)$ must equal $2x + 1 + 4$, or $2x + 5$. Therefore, $f(2) = 2(2) + 5 = 9$.

STRATEGY

What may seem at first to be a complex question could boil down to straightforward substitution.

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Interpreting and Analyzing More Complex Equations in Context

Equations and functions that describe real-life contexts can be complex. Often, it's not possible to analyze them as completely as you can analyze a linear equation or function. You still can acquire key information about the context by interpreting and analyzing the equation or function that describes it. Advanced Math questions may ask you to identify connections between the function, its graph, and the context it describes. You may be asked to use an equation describing a context to determine how a change in one quantity affects another quantity. You may also be asked to manipulate an equation to isolate a quantity of interest on one side of the equation. You may be asked to produce or identify a form of an equation that reveals new information about the context it represents or about the graphical representation of the equation.

Advanced Math Sample Question 16

For a certain reservoir, the function f gives the water level $f(n)$, to the nearest whole percent of capacity, on the nth day of 2016. Which of the following is the best interpretation of $f(37) = 70$?

A) The water level of the reservoir was at 37% capacity for 70 days in 2016.
B) The water level of the reservoir was at 70% capacity for 37 days in 2016.
C) On the 37th day of 2016, the water level of the reservoir was at 70% capacity.
D) On the 70th day of 2016, the water level of the reservoir was at 37% capacity.

The function f gives the water level, to the nearest whole percent of capacity, on the nth day of 2016. It follows that $f(37) = 70$ means that on the 37th day of 2016, the water level of the reservoir was at 70% capacity. This statement is choice C.

Advanced Math Sample Question 17

If an object of mass m is moving at speed v, the object's kinetic energy (KE) is given by the equation $KE = \frac{1}{2}mv^2$. If the mass of the object is halved and its speed is doubled, how does the kinetic energy change?

A) The kinetic energy is halved.
B) The kinetic energy is unchanged.
C) The kinetic energy is doubled.
D) The kinetic energy is quadrupled (multiplied by a factor of 4).

Choice C is correct. If the mass of the object is halved, the new mass is $\frac{m}{2}$. If the speed of the object is doubled, its new speed is $2v$. Therefore, the new kinetic energy is $\frac{1}{2}\left(\frac{m}{2}\right)(2v^2) = \frac{1}{2}\left(\frac{m}{2}\right)(4v^2) = mv^2$. This is double the original kinetic energy of the object, which was $\frac{1}{2}mv^2$.

STRATEGY

Another way to check your answer in sample 17 is to substitute simple numerical values for the variables m, v, and KE when those values are altered as indicated by the question. If the value 1 is substituted for both m and v, then KE is $\frac{1}{2}$. However, substituting $\frac{1}{2}$ for m and 2 for v yields a value for KE of 1. Since 1 is twice the value of $\frac{1}{2}$, you know that KE is doubled.

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Advanced Math Sample Question 18

A gas in a container will escape through holes of microscopic size, as long as the holes are larger than the gas molecules. This process is called effusion. If a gas of molar mass M_1 effuses at a rate of r_1 and a gas of molar mass M_2 effuses at a rate of r_2, then the following relationship holds.

$$\frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}}$$

This is known as Graham's law. Which of the following correctly expresses M_2 in terms of M_1, r_1, and r_2?

A) $M_2 = M_1\left(\frac{r_1^2}{r_2^2}\right)$

B) $M_2 = M_1\left(\frac{r_2^2}{r_1^2}\right)$

C) $M_2 = \sqrt{M_1}\left(\frac{r_1}{r_2}\right)$

D) $M_2 = \sqrt{M_1}\left(\frac{r_2}{r_1}\right)$

Squaring each side of $\frac{r_1}{r_2} = \sqrt{\frac{M_2}{M_1}}$ gives $\left(\frac{r_1}{r_2}\right)^2 = \left(\sqrt{\frac{M_2}{M_1}}\right)^2$, which can be rewritten as $\frac{M_2}{M_1} = \frac{r_1^2}{r_2^2}$. Multiplying each side of $\frac{M_2}{M_1} = \frac{r_1^2}{r_2^2}$ by M_1 gives $M_2 = M_1\left(\frac{r_1^2}{r_2^2}\right)$, which is choice A.

STRATEGY

Always start by identifying exactly what the question asks. In sample 18, you're being asked to isolate the variable M_2. Squaring both sides of the equation is a great first step, as it allows you to eliminate the radical sign.

Advanced Math Sample Question 19

A store manager estimates that if a video game is sold at a price of p dollars, the store will have weekly revenue, in dollars, of $r(p) = -4p^2 + 200p$ from the sale of the video game. Which of the following equivalent forms of $r(p)$ shows, as constants or coefficients, the maximum possible weekly revenue and the price that results in the maximum revenue?

A) $r(p) = 200p - 4p^2$

B) $r(p) = -2(2p^2 - 100p)$

C) $r(p) = -4(p^2 - 50p)$

D) $r(p) = -4(p - 25)^2 + 2{,}500$

Choice D is correct. The graph of r in the coordinate plane is a parabola that opens downward. The maximum value of revenue corresponds to the vertex of the parabola. Since the square of any real number is always nonnegative, the form $r(p) = -4(p - 25)^2 + 2{,}500$ shows that the vertex of the parabola is (25, 2,500); that is, the maximum must occur where $-4(p - 25)^2$ is 0, which is $p = 25$, and this maximum is $r(25) = 2{,}500$. Thus, the maximum possible weekly revenue and the price that results in the maximum revenue occur as constants in the form $r(p) = -4(p - 25)^2 + 2{,}500$.

STRATEGY

The fact that the coefficient of the squared term is negative for this function indicates that the graph of r in the coordinate plane is a parabola that opens downward. Thus, the maximum value of revenue corresponds to the vertex of the parabola.

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Summary: Advanced Math

Main purpose: To assess your understanding of nonlinear relationships

Proportion of the test section: About 35 percent (thirteen to fifteen questions)

Question types

- Equivalent expressions
- Nonlinear equations in one variable and systems of equations in two variables
- Nonlinear functions

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CHAPTER 24

Math: Questions—Advanced Math Drills

1

Which of the following expressions is equivalent to $2(ab - 3) + 2$?

A) $2ab - 1$

B) $2ab - 4$

C) $2ab - 5$

D) $2ab - 8$

2

$$f(x) = (x + 0.25x)(50 - x)$$

The function f is defined as shown. What is the value of $f(20)$?

A) 250

B) 500

C) 750

D) 2,000

3

$$x^2 + 6x + 9 = 36$$

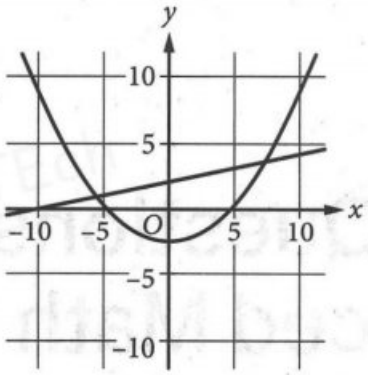
What is the positive solution to the given equation?

4

The function f is defined by $f(x) = \frac{22}{x+1}$. What is the value of $f(22)$?

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5



A system of equations consists of a quadratic equation and a linear equation. The equations in this system are graphed in the *xy*-plane shown. How many solutions does this system have?

A) 0

B) 1

C) 2

D) 3

6

$$S = 4\pi r^2$$

The formula shown gives the surface area, S, of a sphere in terms of the length of its radius, r. Which of the following gives the radius of the sphere in terms of its surface area?

A) $r = \sqrt{\frac{S}{4\pi}}$

B) $r = \sqrt{\frac{4\pi}{S}}$

C) $r = \frac{\sqrt{S}}{4\pi}$

D) $r = \frac{\sqrt{4\pi}}{S}$

7

$$\frac{4x}{2(x^2 - 1)} - \frac{3x}{3(x^2 - 1)}$$

Which of the following is equivalent to the given expression for $x \neq -1$ and $x \neq 1$?

A) $\frac{1}{6(x - 1)}$

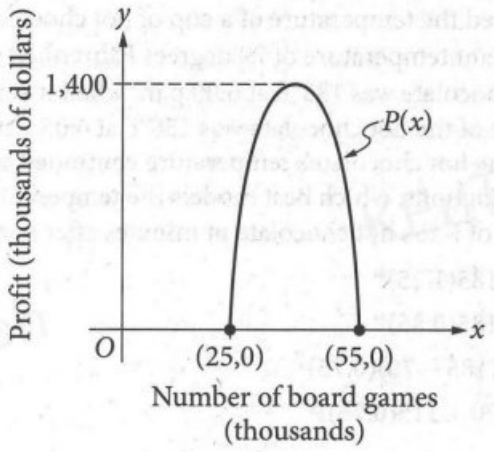
B) $\frac{x}{6(x^2 - 1)}$

C) $\frac{1}{x - 1}$

D) $\frac{x}{x^2 - 1}$

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8



A company produces board games and sells them online and in stores. The quadratic function P models the company's monthly profits $P(x)$, in thousands of dollars, when x board games, in thousands, are produced and sold. The graph of $y = P(x)$, where $25 \leq x \leq 55$, is shown in the xy-plane. How many board games must the company produce and sell in order to earn the maximum profit estimated by the model?

A) 20,000

B) 40,000

C) 60,000

D) 1,400,000

9

$$y - x = 30$$

$$y = x^2 - 28x$$

The graphs of the equations in the given system intersect at the point (x, y) in the xy-plane. What is a possible value of y?

10

In the equation $9x^2 + 108x + \frac{c}{4} = 0$, c is a constant. If the equation has exactly one real solution, what is the value of c?

A) 0

B) 324

C) 1,296

D) 11,664

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11

Kao measured the temperature of a cup of hot chocolate placed in a room with a constant temperature of 70 degrees Fahrenheit (°F). The temperature of the hot chocolate was 185°F at 6:00 p.m. when it started cooling. The temperature of the hot chocolate was 156°F at 6:05 p.m. and 135°F at 6:10 p.m. The hot chocolate's temperature continued to decrease. Of the following functions, which best models the temperature $T(m)$, in degrees Fahrenheit, of Kao's hot chocolate m minutes after it started cooling?

A) $T(m) = 185(1.25)^m$

B) $T(m) = 185(0.85)^m$

C) $T(m) = (185 - 70)(0.75)^{\frac{m}{5}}$

D) $T(m) = 70 + 115(0.75)^{\frac{m}{5}}$

12

$$\sqrt{5(x-k)} = x - k$$

In the given equation, k is a positive constant. The greatest solution to the equation is 12. What is the value of k?

13

$$g(x) = (5 - 2x)(14 + 2x)$$

The function g is defined by the given equation. For what value of x does $g(x)$ reach its maximum?

14

A rectangular volleyball court has an area of 162 square meters. If the length of the court is twice the width, what is the width of the court, in meters?

A) 9

B) 18

C) 27

D) 54

15

$$x^2 = 6x + y$$

$$y = -6x + 36$$

A solution to the given system of equations is (x, y). Which of the following is a possible value of xy?

A) 0

B) 6

C) 12

D) 36

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16

The function f is defined by $f(x) = ax^2 + bx + c$, where a, b, and c are constants and $1 < a < 4$. The graph of $y = f(x)$ in the xy-plane passes through points $(11, 0)$ and $(-2, 0)$. If a is an integer, what could be the value of $a + b$?

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