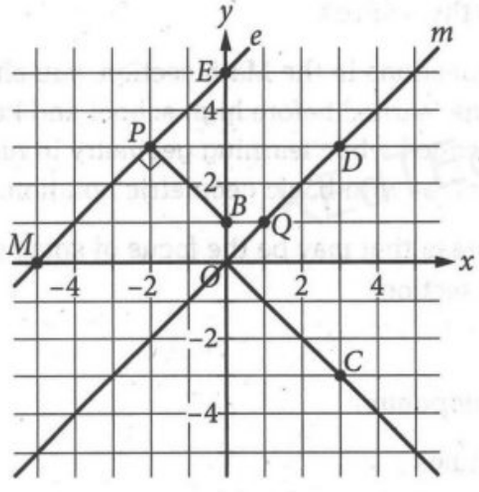
REMINDER

The triangle inequality theorem states that for any triangle, the length of any side of the triangle must be less than the sum of the lengths of the other two sides of the triangle and greater than the difference of the lengths of the other two sides.

- Triangles and other polygons
  - Right triangles and the Pythagorean theorem
  - Properties of equilateral and isosceles triangles
  - Properties of 30°-60°-90° triangles and 45°-45°-90° triangles
  - Congruent triangles and other congruent figures
  - Similar triangles and other similar figures
  - The triangle inequality theorem
  - Squares, rectangles, parallelograms, trapezoids, and other quadrilaterals
  - Regular polygons
- Circles
  - Radius, diameter, and circumference
  - Measure of central angles and inscribed angles
  - Arc length, arc measure, and area of sectors
  - Tangents and chords
  - Representations in the *xy*-plane
- Area and volume
  - Area of plane figures
  - Volume of solids
  - Surface area of solids

You should be familiar with the geometric notation for points and lines, line segments, angles and their measures, and lengths.



In the figure shown, the *xy*-plane has origin *O*. The values of *x* on the horizontal *x*-axis increase as you move to the right, and the values of *y* on the vertical *y*-axis increase as you move up. Line *e* contains point *P*, which has coordinates (−2, 3); point *E*, which has coordinates (0, 5); and point *M*, which has coordinates (−5, 0). Line *m* passes through the origin *O* (0, 0), the point *Q* (1, 1), and the point *D* (3, 3).

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Lines e and m are parallel—they never meet. This is written $e \parallel m$.

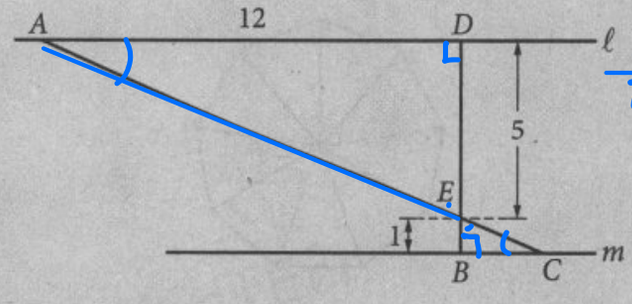
You'll also need to know the following notation:

- $\overleftrightarrow{PE}$: The line containing the points P and E (this is the same as line e)
- $\overline{PE}$, or line segment PE: The line segment with endpoints P and E
- PE: The length of segment PE (you can write $PE = 2\sqrt{2}$)
- $\overrightarrow{PE}$: The ray starting at point P and extending indefinitely in the direction of point E
- $\overrightarrow{EP}$: The ray starting at point E and extending indefinitely in the direction of point P
- $\angle DOC$: The angle formed by $\overrightarrow{OD}$ and $\overrightarrow{OC}$
- $\triangle PEB$: The triangle with vertices P, E, and B
- Quadrilateral $BPMO$: The quadrilateral with vertices B, P, M, and O
- $\overline{BP} \perp \overline{PM}$: Segment BP is perpendicular to segment PM. (You should also recognize that the right-angle box within $\angle BPM$ means this angle is a right angle.)

STRATEGY

Familiarize yourself with these notations in order to avoid confusion on test day.

Geometry and Trigonometry Sample Question 1



In the figure shown, line ℓ is parallel to line m, segment BD is perpendicular to line m, and segment AC and segment BD intersect at E. What is the length of segment AC?

$\triangle ADE \sim \triangle EBC$

$\frac{13}{EC} = \frac{AE}{EC} = \frac{DE}{EB} = \frac{5}{1}$

$13 = 5EC \quad EC = \frac{13}{5}$

$13 + \frac{13}{5} = \frac{65+13}{5}$

$= \frac{78}{5}$

Note that this sample question has no answer choices. It's in the student-produced response (SPR) format. You'll have to come up with your own answer.

Since segment AC and segment BD intersect at E, $\angle AED$ and $\angle CEB$ are vertical angles, and so the measure of $\angle AED$ is equal to the measure of $\angle CEB$. Since line ℓ is parallel to line m, $\angle BCE$ and $\angle DAE$ are alternate interior angles of parallel lines cut by a transversal, and so the measure of $\angle BCE$ is equal to the measure of $\angle DAE$. By the angle-angle similarity theorem, $\triangle AED$ is similar to $\triangle CEB$, with vertices A, E, and D corresponding to vertices C, E, and B, respectively.

Also, $\triangle AED$ is a right triangle, so by the Pythagorean theorem, $AE = \sqrt{AD^2 + DE^2}$. Substituting values for the length of the line segments gives $\sqrt{12^2 + 5^2} = \sqrt{169}$, which means that $AE = 13$.

STRATEGY

A shortcut here is remembering that 5, 12, 13 is a Pythagorean triple (5 and 12 are the lengths of the sides of the right triangle, and 13 is the length of the hypotenuse). Another common Pythagorean triple is 3, 4, 5.

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STRATEGY

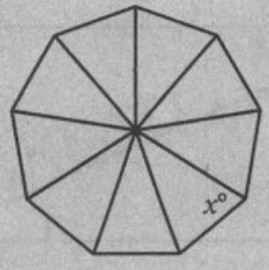
Note how sample 1 requires the knowledge and application of numerous fundamental geometry concepts. Develop mastery of the fundamental concepts, and practice applying them on test-like questions.

Since $\triangle AED$ is similar to $\triangle CEB$, the ratios of the lengths of corresponding sides of the two triangles are in the same proportion. Since $\frac{ED}{EB} = \frac{AE}{EC}$, then $\frac{5}{1} = \frac{13}{EC}$, and so $EC = \frac{13}{5}$. Therefore, $AC = AE + EC$, or $13 + \frac{13}{5} = \frac{78}{5}$.

Note some of the key concepts that were used in sample 1:

- Vertical angles have the same measure.
- When parallel lines are cut by a transversal, the alternate interior angles have the same measure.
- If two angles of a triangle are congruent to (have the same measure as) two angles of another triangle, the two triangles are similar.
- The Pythagorean theorem: $a^2 + b^2 = c^2$, where a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse.
- If two triangles are similar, then all ratios of lengths of corresponding sides are equal.
- If point E lies on line segment AC, then $AC = AE + EC$.

Geometry and Trigonometry Sample Question 2



70°

In the figure shown, a regular polygon with 9 sides has been divided into 9 congruent isosceles triangles by line segments drawn from the center of the polygon to its vertices. What is the value of x?

The sum of the measures of the angles around a point is 360°. Since the 9 triangles are congruent, the measures of each of the 9 angles around the center point are equal. Thus, the measure of each of the 9 angles around the center point is $\frac{360°}{9} = 40°$. In any triangle, the sum of the measures of the interior angles is 180°. So in each triangle, the sum of the measures of the remaining two angles is $180° - 40° = 140°$. Since each triangle is isosceles, the measure of each of these two angles is the same. Therefore, the measure of each of these angles is $\frac{140°}{2} = 70°$. Hence, the value of x is 70.

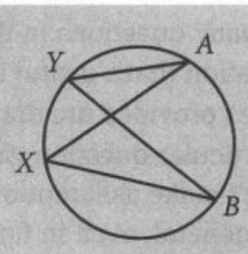
Note some of the key concepts that were used in sample 2:

- The sum of the measures of the angles about a point is 360°.
- Corresponding angles of congruent triangles have the same measure.

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- The sum of the measures of the interior angles of any triangle is 180°.
- In an isosceles triangle, the angles opposite the sides of equal length are of equal measure.

Geometry and Trigonometry Sample Question 3



In the figure shown, $\angle AXB$ and $\angle AYB$ are inscribed in the circle. Which of the following statements is true?

A) The measure of $\angle AXB$ is greater than the measure of $\angle AYB$.
B) The measure of $\angle AXB$ is less than the measure of $\angle AYB$.
C) The measure of $\angle AXB$ is equal to the measure of $\angle AYB$.
D) There is not enough information to determine the relationship between the measure of $\angle AXB$ and the measure of $\angle AYB$.

Choice C is correct. Let the measure of arc AB be $d°$. Since $\angle AXB$ is inscribed in the circle and intercepts arc AB, the measure of $\angle AXB$ is equal to half the measure of arc AB. Thus, the measure of $\angle AXB$ is $\frac{d°}{2}$. Similarly, since $\angle AYB$ is also inscribed in the circle and intercepts arc AB, the measure of $\angle AYB$ is also $\frac{d°}{2}$. Therefore, the measure of $\angle AXB$ is equal to the measure of $\angle AYB$.

STRATEGY

At first glance, it may appear as though there's not enough information to determine the relationship between the two angle measures. One key to this question is identifying what's the same about the two angle measures. In this case, both angles intercept arc AB.

Note the key concept that was used in sample 3:

- The measure of an angle inscribed in a circle is equal to half the measure of its intercepted arc.

You also should know these related concepts:

- The measure of a central angle in a circle is equal to the measure of its intercepted arc.
- An arc is measured in degrees, while arc length is measured in linear units.

You should also be familiar with notation for arcs and circles for questions in the Math section.

- A circle may be identified by the point at its center—for instance, "the circle centered at point M" or "the circle with center at point M."
- An arc named with only its two endpoints, such as $\overset{\frown}{AB}$ or arc AB, will always refer to a minor arc. A minor arc has a measure that's less than 180°.

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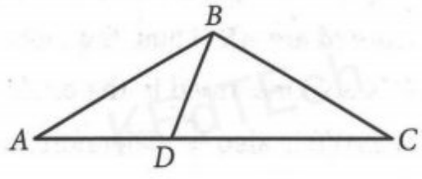
- An arc may also be named with three points: the two endpoints and a third point that the arc passes through. So $\overset{\frown}{ACB}$ or arc *ACB* has endpoints at *A* and *B* and passes through point *C*. Three points may be used to name a minor arc or an arc that has a measure of 180° or more.

REMINDER

Figures are drawn to scale in the SAT Math section unless explicitly stated otherwise. If a question states that a figure isn't drawn to scale, be careful not to make unwarranted assumptions about the figure.

In general, figures that accompany questions in the Math section are intended to provide information that's useful in answering the question. Sometimes figures provided are drawn to scale, and sometimes they're not. In a particular question when it's stated that the figure isn't drawn to scale, don't make assumptions about the relative size of angles or segments. In general, even in figures not drawn to scale, the relative positions of points and angles may be assumed to be in the order shown. Also, line segments that extend through points and appear to lie on the same line may be assumed to be on the same line. A point that appears to lie on a line or curve may be assumed to lie on the line or curve.

The text "Note: Figure not drawn to scale" is included with the figure when degree measures may not be accurately shown and specific lengths may not be drawn proportionally. The following example illustrates what information can and can't be assumed from a figure not drawn to scale.



Note: Figure not drawn to scale.

A question may refer to a triangle such as *ABC* above. Although the note indicates that the figure isn't drawn to scale, you may assume the following from the figure:

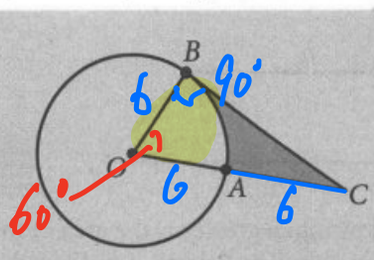
- *ABC*, *ABD*, and *DBC* are triangles.
- *A*, *D*, and *C* are points on a line.
- *D* is between *A* and *C*.
- The length of $\overline{AD}$ is less than the length of $\overline{AC}$.
- The measure of angle *ABD* is less than the measure of angle *ABC*.

You may *not* assume the following from the figure:

- The length of $\overline{AD}$ is less than the length of $\overline{DC}$.
- The measures of angles *BAD* and *DBA* are equal.
- The measure of angle *DBC* is greater than the measure of angle *ABD*.
- Angle *DBC* is a right angle.

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Geometry and Trigonometry Sample Question 4



In the given figure, O is the center of the circle, segment BC is tangent to the circle at B, and A lies on segment OC. If the lengths of $\overline{OB}$ and $\overline{AC}$ are each 6, what is the area of the shaded region?

A) $18\sqrt{3} - 3\pi$
B) $18\sqrt{3} - 6\pi$
C) $36\sqrt{3} - 3\pi$
D) $36\sqrt{3} - 6\pi$

Since line segment BC is tangent to the circle at B, it follows that $\overline{BC} \perp \overline{OB}$, and so triangle OBC is a right triangle with its right angle at B. Since $OB = 6$ and $\overline{OB}$ and $\overline{OA}$ are both radii of the circle, $OA = 6$. Since $OC = OA + AC$, $OC = 6 + 6$, or 12. Thus, triangle OBC is a right triangle with the length of the hypotenuse ($OC = 12$) twice the length of one of its legs ($OB = 6$). It follows that triangle OBC is a 30°-60°-90° triangle with its 30° angle at C and its 60° angle at O. The area of the shaded region is the area of triangle OBC minus the area of the sector bounded by radii OA and OB.

In the 30°-60°-90° triangle OBC, the length of side OB, which is opposite the 30° angle, is 6. Thus, the length of side BC, which is opposite the 60° angle, is $6\sqrt{3}$. Hence, the area of triangle OBC is $\frac{1}{2}(6)(6\sqrt{3}) = 18\sqrt{3}$. Since the sector bounded by radii OA and OB has central angle 60°, the area of this sector is $\frac{60}{360} = \frac{1}{6}$ of the area of the circle. Since the circle has radius 6, its area is $\pi(6)^2 = 36\pi$, and so the area of the sector is $\frac{1}{6}(36\pi) = 6\pi$. Therefore, the area of the shaded region is $18\sqrt{3} - 6\pi$, which is choice B.

Note some of the key concepts that were used in sample 4:

- A tangent to a circle is perpendicular to the radius of the circle drawn to the point of tangency.
- Properties of 30°-60°-90° triangles.
- Area of a circle.
- The area of a sector with central angle $x°$ is equal to $\frac{x}{360}$ of the area of the entire circle.

STRATEGY

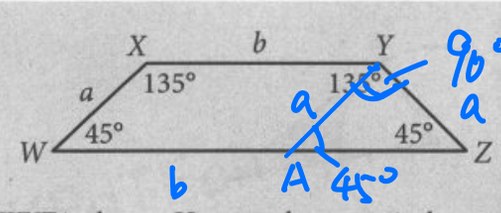
On complex multistep questions such as sample 4, start by identifying the task (finding the area of the shaded region) and considering the intermediate steps that you'll need to solve for (the area of triangle OBC and the area of sector OBA) in order to get to the final answer. Breaking up this question into a series of smaller questions will make it more manageable.

REMINDER

Arc length, area of a sector, and central angle are all proportional to each other in a circle. This proportionality is written as $\frac{\text{arc length}}{\text{circumference}} = \frac{\text{central angle}}{360 \text{ degrees}} = \frac{\text{area of a sector}}{\text{area of a circle}}$.

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Geometry and Trigonometry Sample Question 5



Trapezoid $WXYZ$ is shown. How much greater is the area of this trapezoid than the area of a parallelogram with side lengths a and b and base angles of measures 45° and 135°?

A) $\frac{1}{2}a^2$

B) $\sqrt{2}a^2$

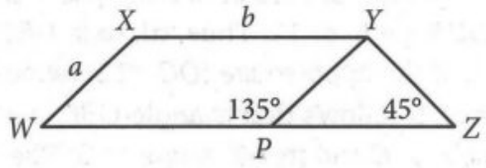
C) $\frac{1}{2}ab$

D) $\sqrt{2}ab$

STRATEGY

Note how drawing the parallelogram within trapezoid $WXYZ$ makes it much easier to compare the areas of the two shapes, minimizing the amount of calculation needed to arrive at the solution. Be on the lookout for time-saving shortcuts such as this one.

In the figure, draw a line segment from Y to the point P on side WZ of the trapezoid such that $\angle YPW$ has measure 135°, as shown in the figure below.



Since in trapezoid $WXYZ$ side XY is parallel to side WZ, it follows that $WXYP$ is a parallelogram with side lengths a and b and base angles of measure 45° and 135°. Thus, the area of the trapezoid is greater than the area of a parallelogram with side lengths a and b and base angles of measure 45° and 135° by the area of triangle PYZ. Since $\angle YPW$ has measure 135°, it follows that $\angle YPZ$ has measure 45°. Hence, triangle PYZ is a 45°-45°-90° triangle with legs of length a. Therefore, its area is $\frac{1}{2}a^2$, which is choice A.

Note some of the key concepts that were used in sample 5:

- Properties of trapezoids and parallelograms.
- Area of a 45°-45°-90° triangle.

Some questions in the Math section may ask you to find the area, surface area, or volume of an object, possibly in context.

Geometry and Trigonometry Sample Question 6

A laboratory supply company produces graduated cylinders, each with an internal radius of 2 inches and an internal height between 7.75 inches and 8 inches. What is one possible volume, rounded to the nearest cubic inch, of a graduated cylinder produced by this company?

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