CHAPTER 27

Math: Questions—Geometry and Trigonometry

In addition to questions in Algebra, Advanced Math, and Problem-Solving and Data Analysis, the Math section includes several questions that are drawn from geometry and trigonometry. They include both multiple-choice and student-produced response questions.

Let's explore the content and skills assessed by these questions.

Geometry

The Math section includes questions that assess your understanding of the key concepts in the geometry of lines, angles, triangles, circles, and other geometric objects. Other questions may also ask you to find the area, surface area, or volume of an abstract figure or a real-life object. You don't need to memorize a large collection of formulas, but you should be comfortable understanding and using these formulas to solve various types of problems. Many of the geometry formulas are provided in the reference sheet available during testing, and less commonly used formulas required to answer particular questions are given in the questions themselves.

REMINDER

You don't need to memorize a large collection of geometry formulas. Many geometry formulas are provided in the reference sheet available throughout testing.

To answer geometry questions in the Math section, you should recall the geometry definitions learned before high school and know the essential concepts extended while learning geometry in high school. You should also be familiar with basic geometric notation.

Here are some of the areas that may be the focus of some geometry questions in the Math section.

- Lines and angles
  - Lengths and midpoints
  - Measures of angles
  - Vertical angles
  - Angle addition
  - Straight angles and the sum of the angles about a point
  - Properties of parallel lines and the angles formed when parallel lines are cut by a transversal
  - Properties of perpendicular lines

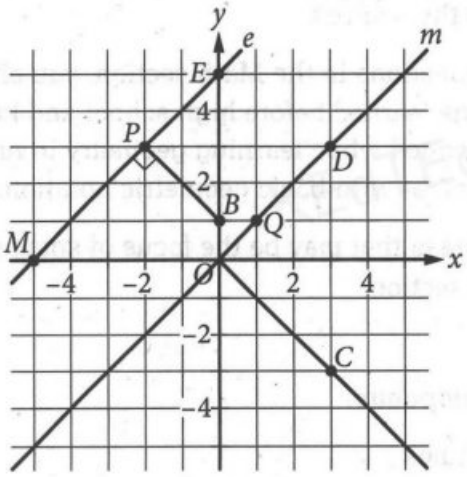
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REMINDER

The triangle inequality theorem states that for any triangle, the length of any side of the triangle must be less than the sum of the lengths of the other two sides of the triangle and greater than the difference of the lengths of the other two sides.

- Triangles and other polygons
  - Right triangles and the Pythagorean theorem
  - Properties of equilateral and isosceles triangles
  - Properties of 30°-60°-90° triangles and 45°-45°-90° triangles
  - Congruent triangles and other congruent figures
  - Similar triangles and other similar figures
  - The triangle inequality theorem
  - Squares, rectangles, parallelograms, trapezoids, and other quadrilaterals
  - Regular polygons
- Circles
  - Radius, diameter, and circumference
  - Measure of central angles and inscribed angles
  - Arc length, arc measure, and area of sectors
  - Tangents and chords
  - Representations in the *xy*-plane
- Area and volume
  - Area of plane figures
  - Volume of solids
  - Surface area of solids

You should be familiar with the geometric notation for points and lines, line segments, angles and their measures, and lengths.



In the figure shown, the *xy*-plane has origin *O*. The values of *x* on the horizontal *x*-axis increase as you move to the right, and the values of *y* on the vertical *y*-axis increase as you move up. Line *e* contains point *P*, which has coordinates (−2, 3); point *E*, which has coordinates (0, 5); and point *M*, which has coordinates (−5, 0). Line *m* passes through the origin *O* (0, 0), the point *Q* (1, 1), and the point *D* (3, 3).

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Lines e and m are parallel—they never meet. This is written $e \parallel m$.

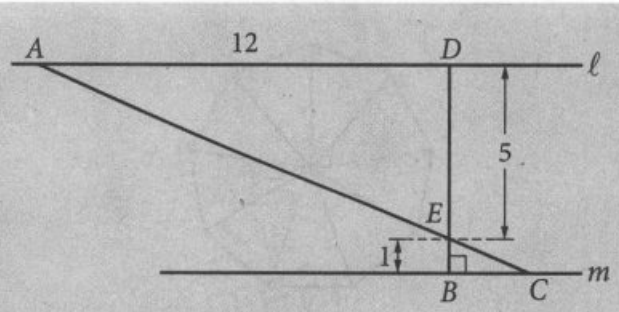
You'll also need to know the following notation:

- $\overleftrightarrow{PE}$: The line containing the points P and E (this is the same as line e)
- $\overline{PE}$, or line segment PE: The line segment with endpoints P and E
- PE: The length of segment PE (you can write $PE = 2\sqrt{2}$)
- $\overrightarrow{PE}$: The ray starting at point P and extending indefinitely in the direction of point E
- $\overrightarrow{EP}$: The ray starting at point E and extending indefinitely in the direction of point P
- $\angle DOC$: The angle formed by $\overrightarrow{OD}$ and $\overrightarrow{OC}$
- $\triangle PEB$: The triangle with vertices P, E, and B
- Quadrilateral $BPMO$: The quadrilateral with vertices B, P, M, and O
- $\overline{BP} \perp \overline{PM}$: Segment BP is perpendicular to segment PM. (You should also recognize that the right-angle box within $\angle BPM$ means this angle is a right angle.)

STRATEGY

Familiarize yourself with these notations in order to avoid confusion on test day.

Geometry and Trigonometry Sample Question 1



In the figure shown, line ℓ is parallel to line m, segment BD is perpendicular to line m, and segment AC and segment BD intersect at E. What is the length of segment AC?

Note that this sample question has no answer choices. It's in the student-produced response (SPR) format. You'll have to come up with your own answer.

Since segment AC and segment BD intersect at E, $\angle AED$ and $\angle CEB$ are vertical angles, and so the measure of $\angle AED$ is equal to the measure of $\angle CEB$. Since line ℓ is parallel to line m, $\angle BCE$ and $\angle DAE$ are alternate interior angles of parallel lines cut by a transversal, and so the measure of $\angle BCE$ is equal to the measure of $\angle DAE$. By the angle-angle similarity theorem, $\triangle AED$ is similar to $\triangle CEB$, with vertices A, E, and D corresponding to vertices C, E, and B, respectively.

Also, $\triangle AED$ is a right triangle, so by the Pythagorean theorem, $AE = \sqrt{AD^2 + DE^2}$. Substituting values for the length of the line segments gives $\sqrt{12^2 + 5^2} = \sqrt{169}$, which means that $AE = 13$.

STRATEGY

A shortcut here is remembering that 5, 12, 13 is a Pythagorean triple (5 and 12 are the lengths of the sides of the right triangle, and 13 is the length of the hypotenuse). Another common Pythagorean triple is 3, 4, 5.

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Since $\triangle AED$ is similar to $\triangle CEB$, the ratios of the lengths of corresponding sides of the two triangles are in the same proportion. Since $\frac{ED}{EB} = \frac{AE}{EC}$, then $\frac{5}{1} = \frac{13}{EC}$, and so $EC = \frac{13}{5}$. Therefore, $AC = AE + EC$, or $13 + \frac{13}{5} = \frac{78}{5}$.

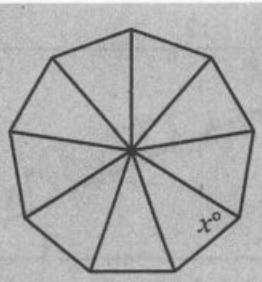
Note some of the key concepts that were used in sample 1:

- Vertical angles have the same measure.
- When parallel lines are cut by a transversal, the alternate interior angles have the same measure.
- If two angles of a triangle are congruent to (have the same measure as) two angles of another triangle, the two triangles are similar.
- The Pythagorean theorem: $a^2 + b^2 = c^2$, where a and b are the lengths of the legs of a right triangle and c is the length of the hypotenuse.
- If two triangles are similar, then all ratios of lengths of corresponding sides are equal.
- If point E lies on line segment AC, then $AC = AE + EC$.

STRATEGY

Note how sample 1 requires the knowledge and application of numerous fundamental geometry concepts. Develop mastery of the fundamental concepts, and practice applying them on test-like questions.

Geometry and Trigonometry Sample Question 2



In the figure shown, a regular polygon with 9 sides has been divided into 9 congruent isosceles triangles by line segments drawn from the center of the polygon to its vertices. What is the value of x?

The sum of the measures of the angles around a point is 360°. Since the 9 triangles are congruent, the measures of each of the 9 angles around the center point are equal. Thus, the measure of each of the 9 angles around the center point is $\frac{360°}{9} = 40°$. In any triangle, the sum of the measures of the interior angles is 180°. So in each triangle, the sum of the measures of the remaining two angles is $180° - 40° = 140°$. Since each triangle is isosceles, the measure of each of these two angles is the same. Therefore, the measure of each of these angles is $\frac{140°}{2} = 70°$. Hence, the value of x is 70.

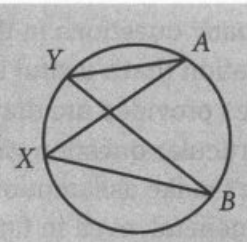
Note some of the key concepts that were used in sample 2:

- The sum of the measures of the angles about a point is 360°.
- Corresponding angles of congruent triangles have the same measure.

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- The sum of the measures of the interior angles of any triangle is 180°.
- In an isosceles triangle, the angles opposite the sides of equal length are of equal measure.

Geometry and Trigonometry Sample Question 3



In the figure shown, $\angle AXB$ and $\angle AYB$ are inscribed in the circle. Which of the following statements is true?

A) The measure of $\angle AXB$ is greater than the measure of $\angle AYB$.
B) The measure of $\angle AXB$ is less than the measure of $\angle AYB$.
C) The measure of $\angle AXB$ is equal to the measure of $\angle AYB$.
D) There is not enough information to determine the relationship between the measure of $\angle AXB$ and the measure of $\angle AYB$.

Choice C is correct. Let the measure of arc AB be $d°$. Since $\angle AXB$ is inscribed in the circle and intercepts arc AB, the measure of $\angle AXB$ is equal to half the measure of arc AB. Thus, the measure of $\angle AXB$ is $\frac{d°}{2}$. Similarly, since $\angle AYB$ is also inscribed in the circle and intercepts arc AB, the measure of $\angle AYB$ is also $\frac{d°}{2}$. Therefore, the measure of $\angle AXB$ is equal to the measure of $\angle AYB$.

STRATEGY

At first glance, it may appear as though there's not enough information to determine the relationship between the two angle measures. One key to this question is identifying what's the same about the two angle measures. In this case, both angles intercept arc AB.

Note the key concept that was used in sample 3:

- The measure of an angle inscribed in a circle is equal to half the measure of its intercepted arc.

You also should know these related concepts:

- The measure of a central angle in a circle is equal to the measure of its intercepted arc.
- An arc is measured in degrees, while arc length is measured in linear units.

You should also be familiar with notation for arcs and circles for questions in the Math section.

- A circle may be identified by the point at its center—for instance, "the circle centered at point M" or "the circle with center at point M."
- An arc named with only its two endpoints, such as $\overset{\frown}{AB}$ or arc AB, will always refer to a minor arc. A minor arc has a measure that's less than 180°.

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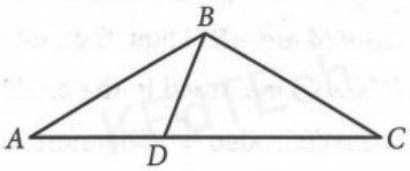
- An arc may also be named with three points: the two endpoints and a third point that the arc passes through. So $\overset{\frown}{ACB}$ or arc *ACB* has endpoints at *A* and *B* and passes through point *C*. Three points may be used to name a minor arc or an arc that has a measure of 180° or more.

REMINDER

Figures are drawn to scale in the SAT Math section unless explicitly stated otherwise. If a question states that a figure isn't drawn to scale, be careful not to make unwarranted assumptions about the figure.

In general, figures that accompany questions in the Math section are intended to provide information that's useful in answering the question. Sometimes figures provided are drawn to scale, and sometimes they're not. In a particular question when it's stated that the figure isn't drawn to scale, don't make assumptions about the relative size of angles or segments. In general, even in figures not drawn to scale, the relative positions of points and angles may be assumed to be in the order shown. Also, line segments that extend through points and appear to lie on the same line may be assumed to be on the same line. A point that appears to lie on a line or curve may be assumed to lie on the line or curve.

The text "Note: Figure not drawn to scale" is included with the figure when degree measures may not be accurately shown and specific lengths may not be drawn proportionally. The following example illustrates what information can and can't be assumed from a figure not drawn to scale.



Note: Figure not drawn to scale.

A question may refer to a triangle such as *ABC* above. Although the note indicates that the figure isn't drawn to scale, you may assume the following from the figure:

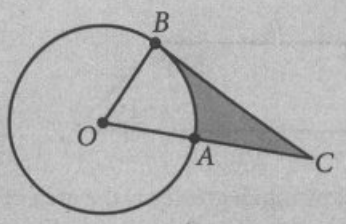
- *ABC*, *ABD*, and *DBC* are triangles.
- *A*, *D*, and *C* are points on a line.
- *D* is between *A* and *C*.
- The length of $\overline{AD}$ is less than the length of $\overline{AC}$.
- The measure of angle *ABD* is less than the measure of angle *ABC*.

You may *not* assume the following from the figure:

- The length of $\overline{AD}$ is less than the length of $\overline{DC}$.
- The measures of angles *BAD* and *DBA* are equal.
- The measure of angle *DBC* is greater than the measure of angle *ABD*.
- Angle *DBC* is a right angle.

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Geometry and Trigonometry Sample Question 4



In the given figure, O is the center of the circle, segment BC is tangent to the circle at B, and A lies on segment OC. If the lengths of $\overline{OB}$ and $\overline{AC}$ are each 6, what is the area of the shaded region?

A) $18\sqrt{3} - 3\pi$
B) $18\sqrt{3} - 6\pi$
C) $36\sqrt{3} - 3\pi$
D) $36\sqrt{3} - 6\pi$

Since line segment BC is tangent to the circle at B, it follows that $\overline{BC} \perp \overline{OB}$, and so triangle OBC is a right triangle with its right angle at B. Since $OB = 6$ and $\overline{OB}$ and $\overline{OA}$ are both radii of the circle, $OA = 6$. Since $OC = OA + AC$, $OC = 6 + 6$, or 12. Thus, triangle OBC is a right triangle with the length of the hypotenuse ($OC = 12$) twice the length of one of its legs ($OB = 6$). It follows that triangle OBC is a 30°-60°-90° triangle with its 30° angle at C and its 60° angle at O. The area of the shaded region is the area of triangle OBC minus the area of the sector bounded by radii OA and OB.

In the 30°-60°-90° triangle OBC, the length of side OB, which is opposite the 30° angle, is 6. Thus, the length of side BC, which is opposite the 60° angle, is $6\sqrt{3}$. Hence, the area of triangle OBC is $\frac{1}{2}(6)(6\sqrt{3}) = 18\sqrt{3}$. Since the sector bounded by radii OA and OB has central angle 60°, the area of this sector is $\frac{60}{360} = \frac{1}{6}$ of the area of the circle. Since the circle has radius 6, its area is $\pi(6)^2 = 36\pi$, and so the area of the sector is $\frac{1}{6}(36\pi) = 6\pi$. Therefore, the area of the shaded region is $18\sqrt{3} - 6\pi$, which is choice B.

Note some of the key concepts that were used in sample 4:

- A tangent to a circle is perpendicular to the radius of the circle drawn to the point of tangency.
- Properties of 30°-60°-90° triangles.
- Area of a circle.
- The area of a sector with central angle $x°$ is equal to $\frac{x}{360}$ of the area of the entire circle.

STRATEGY

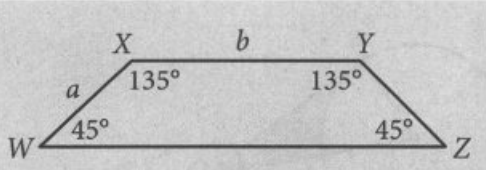
On complex multistep questions such as sample 4, start by identifying the task (finding the area of the shaded region) and considering the intermediate steps that you'll need to solve for (the area of triangle OBC and the area of sector OBA) in order to get to the final answer. Breaking up this question into a series of smaller questions will make it more manageable.

REMINDER

Arc length, area of a sector, and central angle are all proportional to each other in a circle. This proportionality is written as $\frac{\text{arc length}}{\text{circumference}} = \frac{\text{central angle}}{\text{360 degrees}} = \frac{\text{area of a sector}}{\text{area of a circle}}$.

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Geometry and Trigonometry Sample Question 5



Trapezoid $WXYZ$ is shown. How much greater is the area of this trapezoid than the area of a parallelogram with side lengths a and b and base angles of measures 45° and 135°?

A) $\frac{1}{2}a^2$

B) $\sqrt{2}a^2$

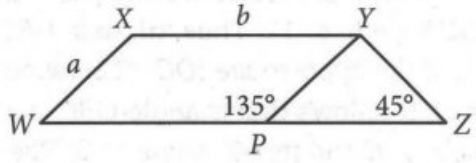
C) $\frac{1}{2}ab$

D) $\sqrt{2}ab$

STRATEGY

Note how drawing the parallelogram within trapezoid $WXYZ$ makes it much easier to compare the areas of the two shapes, minimizing the amount of calculation needed to arrive at the solution. Be on the lookout for time-saving shortcuts such as this one.

In the figure, draw a line segment from Y to the point P on side WZ of the trapezoid such that $\angle YPW$ has measure 135°, as shown in the figure below.



Since in trapezoid $WXYZ$ side XY is parallel to side WZ, it follows that $WXYP$ is a parallelogram with side lengths a and b and base angles of measure 45° and 135°. Thus, the area of the trapezoid is greater than the area of a parallelogram with side lengths a and b and base angles of measure 45° and 135° by the area of triangle PYZ. Since $\angle YPW$ has measure 135°, it follows that $\angle YPZ$ has measure 45°. Hence, triangle PYZ is a 45°-45°-90° triangle with legs of length a. Therefore, its area is $\frac{1}{2}a^2$, which is choice A.

Note some of the key concepts that were used in sample 5:

- Properties of trapezoids and parallelograms.
- Area of a 45°-45°-90° triangle.

Some questions in the Math section may ask you to find the area, surface area, or volume of an object, possibly in context.

Geometry and Trigonometry Sample Question 6

A laboratory supply company produces graduated cylinders, each with an internal radius of 2 inches and an internal height between 7.75 inches and 8 inches. What is one possible volume, rounded to the nearest cubic inch, of a graduated cylinder produced by this company?

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The volume of a cylinder can be found by using the formula $V = \pi r^2 h$, where r is the radius of the circular base and h is the height of the cylinder. The smallest possible volume, in cubic inches, of a graduated cylinder produced by the laboratory supply company can be found by substituting 2 for r and 7.75 for h, giving $V = \pi(2^2)(7.75)$. This gives a volume of approximately 97.39 cubic inches, which rounds to 97 cubic inches. The largest possible volume, in cubic inches, can be found by substituting 2 for r and 8 for h, giving $V = \pi(2^2)(8)$. This gives a volume of approximately 100.53 cubic inches, which rounds to 101 cubic inches. Therefore, the possible volumes are all the integers greater than or equal to 97 and less than or equal to 101, which are 97, 98, 99, 100, and 101. Any of these numbers may be entered as the correct answer.

Coordinate Geometry

Questions in the Geometry and Trigonometry domain may ask you to use the coordinate plane and equations of lines and circles to describe figures. You may be asked to create the equation of a circle given the figure or use the structure of a given equation to determine a property of a figure in the coordinate plane. You should know that the graph of $(x - a)^2 + (y - b)^2 = r^2$ in the xy-plane is a circle with center (a, b) and radius r.

Geometry and Trigonometry Sample Question 7

$$x^2 + (y + 1)^2 = 4$$

The graph of the given equation in the xy-plane is a circle. A new circle is drawn whose center is 1 unit up from this circle's center and whose radius is 1 more than this circle's radius. Which of the following is an equation of this new circle?

A) $x^2 + y^2 = 5$

B) $x^2 + y^2 = 9$

C) $x^2 + (y + 2)^2 = 5$

D) $x^2 + (y + 2)^2 = 9$

The graph of the given equation $x^2 + (y + 1)^2 = 4$ in the xy-plane is a circle with center $(0, -1)$ and radius $\sqrt{4} = 2$. If the center of the new circle is 1 unit up from the center of the given circle, the center of the new circle will be $(0, 0)$. If the radius of the new circle is 1 more than the given circle, the radius of the new circle will be 3. Therefore, an equation of the new circle in the xy-plane is $x^2 + y^2 = 3^2 = 9$, so choice B is correct.

Geometry and Trigonometry Sample Question 8

$$x^2 + 8x + y^2 - 6y = 24$$

The graph of the given equation in the xy-plane is a circle. What is the radius of the circle?

REMINDER

You should know that the graph of $(x - a)^2 + (y - b)^2 = r^2$ in the xy-plane is a circle with center (a, b) and radius r. You should also be comfortable finding the center or radius of a circle from an equation not written in "standard form" by using the method of completing the square to rewrite the equation in standard form.

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The given equation isn't in the standard form $(x - a)^2 + (y - b)^2 = r^2$. You can put it in standard form by completing the square. Since the coefficient of x is 8 and the coefficient of y is −6, you can write the equation in terms of $(x + 4)^2$ and $(y - 3)^2$ as follows:

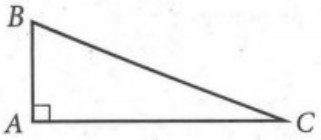
$$x^2 + 8x + y^2 - 6y = 24$$
$$(x^2 + 8x + 16) - 16 + (y^2 - 6y + 9) - 9 = 24$$
$$(x + 4)^2 - 16 + (y - 3)^2 - 9 = 24$$
$$(x + 4)^2 + (y - 3)^2 = 24 + 16 + 9$$
$$(x + 4)^2 + (y - 3)^2 = 49$$

Since $49 = 7^2$, the radius of the circle is 7. (Also, the center of the circle is (−4, 3).)

Trigonometry and Radians

Questions in the Geometry and Trigonometry domain may ask you to apply the definitions from right triangle trigonometry. You should also know the definition of radian measure; you may also need to convert between angle measure in degrees and radians. You may need to evaluate trigonometric functions at benchmark angle measures such as 0, $\frac{\pi}{6}$, $\frac{\pi}{4}$, $\frac{\pi}{3}$, and $\frac{\pi}{2}$ radians (which are equal to the angle measures 0°, 30°, 45°, 60°, and 90°, respectively). You will *not* be asked for values of trigonometric functions that require a calculator.

For an acute angle, the trigonometric functions sine, cosine, and tangent can be defined using right triangles. (Note the functions are often abbreviated as sin, cos, and tan, respectively.)



For $\angle C$ in the right triangle shown:

- $\sin \angle C = \frac{AB}{BC} = \frac{\text{length of leg opposite } \angle C}{\text{length of hypotenuse}}$
- $\cos \angle C = \frac{AC}{BC} = \frac{\text{length of leg adjacent to } \angle C}{\text{length of hypotenuse}}$
- $\tan \angle C = \frac{AB}{AC} = \frac{\text{length of leg opposite } \angle C}{\text{length of leg adjacent to } \angle C} = \frac{\sin \angle C}{\cos \angle C}$

REMINDER

The acronym "SOHCAHTOA" may help you remember how to compute sine, cosine, and tangent. SOH stands for Sine equals Opposite over Hypotenuse, CAH stands for Cosine equals Adjacent over Hypotenuse, and TOA stands for Tangent equals Opposite over Adjacent.

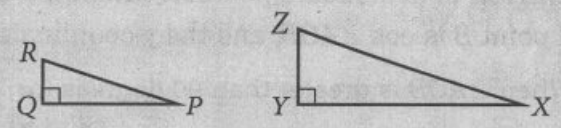
The functions will often be written as sin C, cos C, and tan C, respectively.

Note that the trigonometric functions are actually functions of the *measures* of an angle, not the angle itself. Thus, if the measure of $\angle C$ is, say, 30°, you can write sin 30°, cos 30°, and tan 30°, respectively.

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Also note that sine and cosine are cofunctions and that $\sin B = \frac{\text{length of leg opposite } \angle B}{\text{length of hypotenuse}} = \frac{AC}{BC} = \cos C$. This is the complementary angle relationship: $\sin x° = \cos (90° - x°)$.

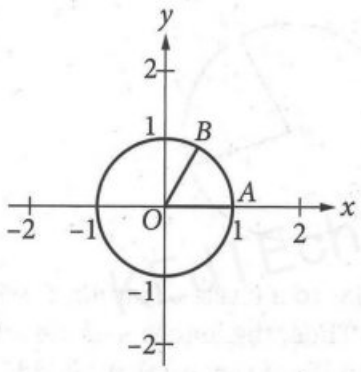
Geometry and Trigonometry Sample Question 9



In the figure, right triangle PQR is similar to right triangle XYZ, with vertices P, Q, and R corresponding to vertices X, Y, and Z, respectively. If $\cos R = 0.263$, what is the value of $\cos Z$?

By the definition of cosine, $\cos R = \frac{RQ}{RP}$ and $\cos Z = \frac{ZY}{ZX}$. Since triangle PQR is similar to triangle XYZ, with vertices P, Q, and R corresponding to vertices X, Y, and Z, respectively, the ratios $\frac{RQ}{RP}$ and $\frac{ZY}{ZX}$ are equal. Therefore, since $\cos R = \frac{RQ}{RP}$, or $\cos R = 0.263$, it follows that $\cos Z = \frac{ZY}{ZX}$, or $\cos Z = 0.263$. Note that to find the values of the trigonometric functions of $d°$, you can use *any* right triangle with an acute angle of measure $d°$ and then find the appropriate ratio of lengths of sides.

Note that since an acute angle of a right triangle has a measure between 0° and 90°, exclusive, right triangles can be used only to find values of trigonometric functions for angles with measures between 0° and 90°, exclusive. The definitions of sine, cosine, and tangent can be extended to all values. This is done using radian measure and the unit circle.



The circle above has radius 1 and is centered on the origin, O. An angle in the coordinate plane is said to be in *standard position* if it meets these two conditions: (1) its vertex lies at the origin and (2) one of its sides lies along the positive x-axis. Since $\angle AOB$, above, formed by segments OA and OB, meets both these conditions, it's in

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standard position. As segment OB, also called the *terminal side* of $\angle AOB$, rotates counterclockwise about the circle, while OA is anchored along the x-axis, the *radian* measure of $\angle AOB$ is defined to be the length s of the arc that $\angle AOB$ intercepts on the unit circle. In other words, the measure of $\angle AOB$ is s radians.

REMINDER

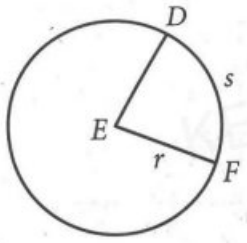
To convert from degrees to radians, multiply the number of degrees by $\frac{2\pi}{360}$ degrees. To convert from radians to degrees, multiply the number of radians by $\frac{360^\circ}{2\pi \text{ radians}}$.

When an acute $\angle AOB$ is in standard position within the unit circle, the x-coordinate of point B is $\cos \angle AOB$, and the y-coordinate of point B is $\sin \angle AOB$. When $\angle AOB$ is greater than 90 degrees (or $\frac{\pi}{2}$ radians), and point B extends beyond the boundaries of the positive x-axis and positive y-axis, the values of $\cos \angle AOB$ and $\sin \angle AOB$ may be negative depending on the coordinates of point B. For any $\angle AOB$, place $\angle AOB$ in standard position within the circle of radius 1 centered at the origin, with side OA along the positive x-axis and terminal side OB intersecting the circle at point B. Then the cosine of $\angle AOB$ is the x-coordinate of B, and the sine of $\angle AOB$ is the y-coordinate of B. The tangent of $\angle AOB$ is the sine of $\angle AOB$ divided by the cosine of $\angle AOB$.

An angle with a full rotation about point O has measure 360°. This angle intercepts the full circumference of the circle, which has length 2π. Thus, $\frac{\text{measure of an angle in radians}}{\text{measure of an angle in degrees}} = \frac{2\pi}{360^\circ}$. It follows that the measure of an angle in radians is $\frac{2\pi}{360^\circ}$ × the measure of the angle in degrees and that the measure of an angle in degrees is $\frac{360^\circ}{2\pi}$ × the measure of the angle in radians.

Also note that since a rotation of 2π about point O brings you back to the same point on the unit circle, $\sin (s + 2\pi) = \sin s$, $\cos (s + 2\pi) = \cos s$, and $\tan (s + 2\pi)$, for any radian measure s.

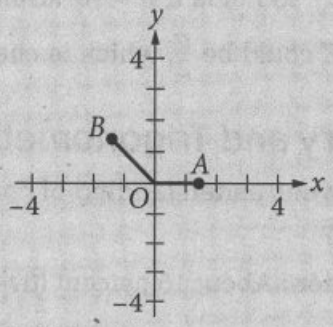
Let angle DEF be a central angle in a circle of radius r, as shown in the following figure.



A circle of radius r is similar to a circle of radius 1, with constant of proportionality equal to r. Thus, the length s of the arc intercepted by angle DEF is r times the length of the arc that would be intercepted by an angle of the same measure in a circle of radius 1. Therefore, in the figure above, $s = r \times$ (radian measure of angle DEF), or radian measure of angle $DEF = \frac{s}{r}$.

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Geometry and Trigonometry Sample Question 10



In the figure shown, the coordinates of point B are $(-\sqrt{2}, \sqrt{2})$. What is the measure, in radians, of angle AOB?

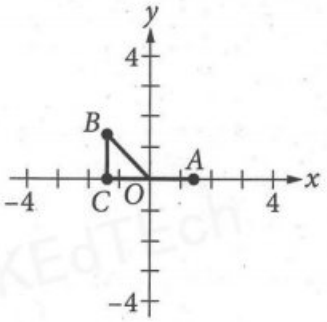
A) $\frac{\pi}{4}$

B) $\frac{\pi}{2}$

C) $\frac{3\pi}{4}$

D) $\frac{5\pi}{4}$

Let C be the point $(-\sqrt{2}, 0)$. Then triangle BOC, shown in the figure below, is a right triangle with both legs of length $\sqrt{2}$.



Hence, triangle BOC is a 45°-45°-90° triangle. Thus, angle COB has measure 45°, and angle AOB has measure 180° − 45° = 135°. Therefore, the measure of angle AOB in radians is $135° \times \frac{2\pi}{360°} = \frac{3\pi}{4}$, which is choice C.

STRATEGY

Always be on the lookout for special right triangles. Here, noticing that segment OB is the hypotenuse of a 45°-45°-90° triangle makes this question easier to solve.

Geometry and Trigonometry Sample Question 11

$$\sin x = \cos (K - x)$$

In the given equation, the angle measures are in radians and K is a constant. Which of the following could be the value of K?

A) 0

B) $\frac{\pi}{4}$

C) $\frac{\pi}{2}$

D) π

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The complementary angle relationship for sine and cosine implies that the equation $\sin x = \cos (K - x)$ holds if $K = 90°$. Since $90° = \frac{2\pi}{360°} \times 90°$, or $\frac{\pi}{2}$ radians, the value of K could be $\frac{\pi}{2}$, which is choice C.

Summary: Geometry and Trigonometry Questions

Main purpose: To assess your understanding of concepts central to geometry and trigonometry

Proportion of the test section: About 15 percent (five to seven questions)

Question types

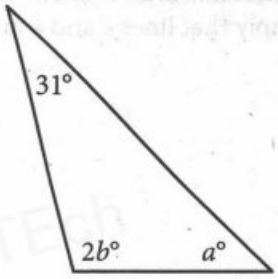
- Area and volume
- Lines, angles, and triangles
- Right triangles and trigonometry
- Circles

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CHAPTER 28

Math: Questions—Geometry and Trigonometry Drills

1



In the triangle shown, $a = 45$. What is the value of b?

A) 52

B) 59

C) 76

D) 104

2

A cube has a surface area of 54 square meters. What is the volume, in cubic meters, of the cube?

A) 18

B) 27

C) 36

D) 81

3

In the xy-plane, a circle with radius 5 has center $(-8, 6)$. Which of the following is an equation of the circle?

A) $(x - 8)^2 + (y + 6)^2 = 25$

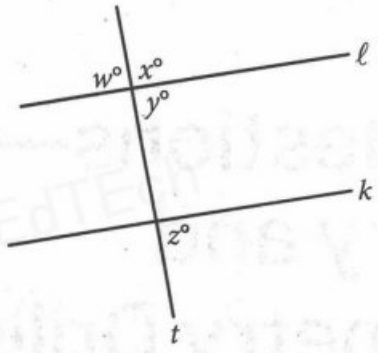
B) $(x + 8)^2 + (y - 6)^2 = 25$

C) $(x - 8)^2 + (y + 6)^2 = 5$

D) $(x + 8)^2 + (y - 6)^2 = 5$

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4



Note: Figure not drawn to scale.

In the figure shown, line t intersects lines ℓ and k. Which of the following statements, if true, would imply that lines ℓ and k are parallel?

A) $w = y$

B) $w = z$

C) $x = z$

D) $x + y = 180$

5

In a right triangle, the tangent of one of the two acute angles is $\frac{\sqrt{3}}{3}$. What is the tangent of the other acute angle?

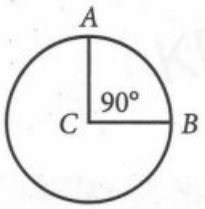
A) $-\frac{\sqrt{3}}{3}$

B) $-\frac{3}{\sqrt{3}}$

C) $\frac{\sqrt{3}}{3}$

D) $\frac{3}{\sqrt{3}}$

6



Point C is the center of the circle shown. What is the measure of angle ACB, in radians?

A) 2π

B) π

C) $\frac{\pi}{2}$

D) $\frac{\pi}{4}$

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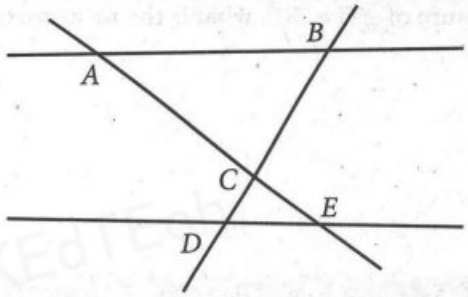
7

The length of one side of square M is 5 times the length of one side of square N. The area of square N is 361 square centimeters. What is the area, in square centimeters, of square M?

8

Triangle KLM is similar to triangle QRS, where angle K corresponds to angle Q and where angles L and R are right angles. If $\sin K = \frac{105}{233}$ and $\sin M = \frac{208}{233}$, what is the value of $\tan S$?

9



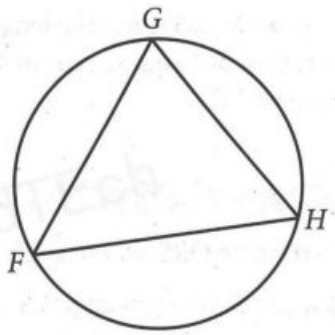
Note: Figure not drawn to scale.

In the figure shown, $\triangle ABC$ is similar to $\triangle EDC$, with $\angle BAC$ corresponding to $\angle CED$ and $\angle ABC$ corresponding to $\angle CDE$. Which of the following must be true?

A) $\overline{AE} \parallel \overline{BD}$
B) $\overline{AE} \perp \overline{BD}$
C) $\overline{AB} \parallel \overline{DE}$
D) $\overline{AB} \perp \overline{DE}$

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10



Note: Figure not drawn to scale.

Triangle *FGH* is inscribed in the circle shown. If arc *FG* is congruent to arc *GH* and the measure of $\angle G$ is 30°, what is the measure of $\angle H$?

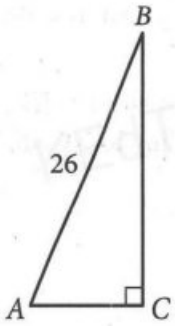
A) 30°

B) 60°

C) 75°

D) 120°

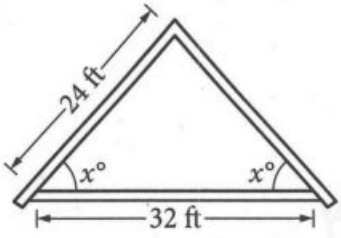
11



Triangle *ABC* shown is a right triangle, and $\sin B = \frac{5}{13}$. What is the length of side $\overline{BC}$?

12

An architect drew the sketch shown while designing a house roof. The dimensions shown are for the interior of the triangle.



Note: Figure not drawn to scale.

What is the value of cos *x*?

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13

Line ℓ is parallel to line m. Points A and B lie on line ℓ, and points P and Q lie on line m. If $\angle ABP$ and $\angle QAB$ each have measure 21° and $\angle AQB$ has measure 76°, what is the measure, in degrees, of $\angle PAQ$?

14



The semicircle shown has a radius of r inches, and chord $\overline{CD}$ is parallel to the diameter $\overline{AB}$. If the length of $\overline{CD}$ is $\frac{2}{3}$ of the length of $\overline{AB}$, what is the distance between the chord and the diameter in terms of r?

A) $\frac{1}{3}\pi r$

B) $\frac{2}{3}\pi r$

C) $\frac{\sqrt{2}}{2}r$

D) $\frac{\sqrt{5}}{3}r$

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