7

Two customers purchased the same kind of bread and eggs at a store. The first customer paid 12.45 dollars for 1 loaf of bread and 2 dozen eggs. The second customer paid 19.42 dollars for 4 loaves of bread and 1 dozen eggs. What is the cost, in dollars, of 1 dozen eggs?

A) 3.77
B) 3.88
C) 4.15
D) 4.34

8

$$y = 18$$
$$y = -3(x - 18)^2 + 15$$

If the given equations are graphed in the xy-plane, at how many points do the graphs of the equations intersect?

A) Exactly one
B) Exactly two
C) Infinitely many
D) Zero

9

$$y = x + 9$$
$$y = x^2 + 16x + 63$$

A solution to the given system of equations is (x, y). What is the greatest possible value of x?

A) -6
B) 7
C) 9
D) 63

10

In the xy-plane, line p has a slope of $-\frac{5}{3}$ and an x-intercept of $(-6, 0)$. What is the y-coordinate of the y-intercept of line p?

Linear equation

11

How many solutions does the equation $12(x - 3) = -3(x + 12)$ have?

A) Exactly one
B) Exactly two
C) Infinitely many
D) Zero

12

Kaylani used fabric measuring 5 yards in length to make each suit for a men's choir. The relationship between the number of suits that Kaylani made, x, and the total length of fabric that she purchased y, in yards, is represented by the equation $y - 5x = 6$. What is the best interpretation of 6 in this context?

A) Kaylani made 6 suits.
B) Kaylani purchased a total of 6 yards of fabric.
C) Kaylani used a total of 6 yards of fabric to make the suits.
D) Kaylani purchased 6 yards more fabric than she used to make the suits.

13

Which quadratic equation has no real solutions?

A) $x^2 + 14x - 49 = 0$
B) $x^2 - 14x + 49 = 0$
C) $5x^2 - 14x - 49 = 0$
D) $5x^2 - 14x + 49 = 0$

$b^2 - 4ac < 0$
Quadratic Formula

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CONTINUE

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14

$$f(x) = (1.84)^{\frac{x}{4}}$$

The function f is defined by the given equation. The equation can be rewritten as $f(x) = \left(1 + \frac{p}{100}\right)^x$, where p is a constant. Which of the following is closest to the value of p?

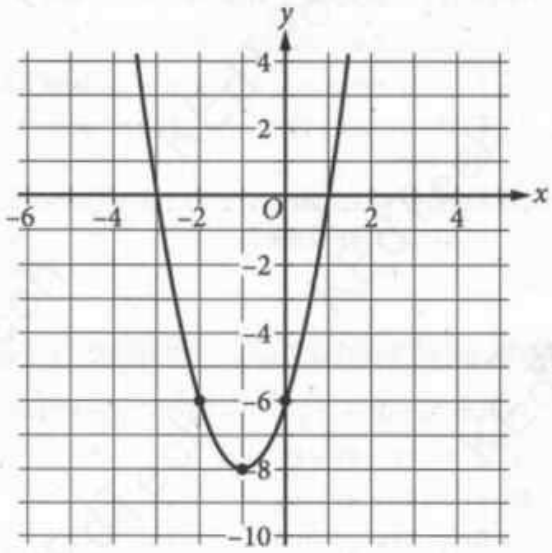
A) 16
B) 21
C) 46
D) 96

15

$$6 + 7r = pw$$
$$7r - 5w = 5w + 11$$

In the given system of equations, p is a constant. If the system has no solution, what is the value of p?

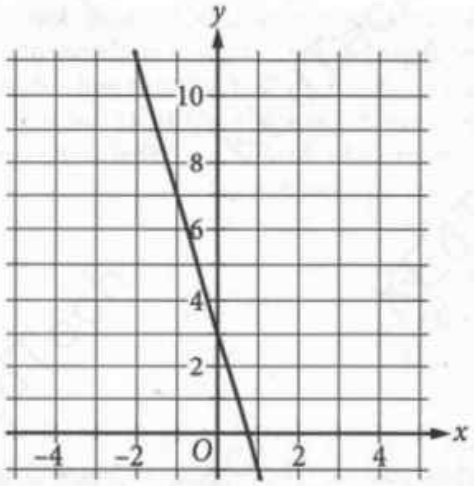
16



The graph of $y = 2x^2 + bx + c$ is shown, where b and c are constants. What is the value of bc?

Quadratic Functions.

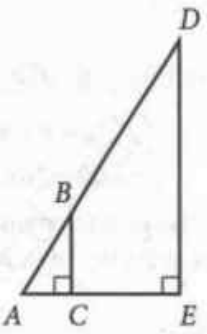
17



The graph of the linear function $y = f(x) + 19$ is shown. If c and d are positive constants, which equation could define f?

A) $f(x) = -d - cx$
B) $f(x) = d - cx$
C) $f(x) = -d + cx$
D) $f(x) = d + cx$

18



Note: Figure not drawn to scale.

In the figure shown, $AB = \sqrt{34}$ units, $AC = 3$ units, and $CE = 21$ units. What is the area, in square units, of triangle ADE?

Similar Triangles

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19

The result of increasing the quantity x by 400% is 60. What is the value of x?

A) 12
B) 15
C) 240
D) 340

$x + \frac{400\%}{100\%}$)

20

In the xy-plane, a circle has center C with coordinates (h, k). Points A and B lie on the circle. Point A has coordinates $(h + 1, k + \sqrt{102})$, and $\angle ACB$ is a right angle. What is the length of $\overline{AB}$?

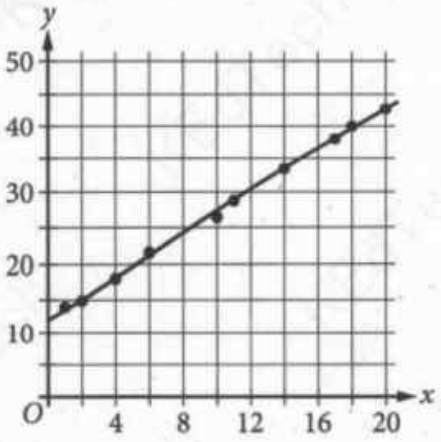
A) $\sqrt{206}$
B) $2\sqrt{102}$
C) $103\sqrt{2}$
D) $103\sqrt{3}$

21

For an electric field passing through a flat surface perpendicular to it, the electric flux of the electric field through the surface is the product of the electric field's strength and the area of the surface. A certain flat surface consists of two adjacent squares, where the side length, in meters, of the larger square is 3 times the side length, in meters, of the smaller square. An electric field with strength 29.00 volts per meter passes uniformly through this surface, which is perpendicular to the electric field. If the total electric flux of the electric field through this surface is 4,640 volts · meters, what is the electric flux, in volts · meters, of the electric field through the larger square?

4176

22



The scatterplot shows the relationship between two variables, x and y, for data set E. A line of best fit is shown. Data set F is created by multiplying the y-coordinate of each data point from data set E by 3.9. Which of the following could be an equation of a line of best fit for data set F?

A) $y = 46.8 + 5.9x$
B) $y = 46.8 + 1.5x$
C) $y = 12 + 5.9x$
D) $y = 12 + 1.5x$

STOP

If you finish before time is called, you may check your work on this module only. Do not turn to any other module in the test.

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$$6 + 7r = pw$$

$$7r - 5w = 5w + 11$$

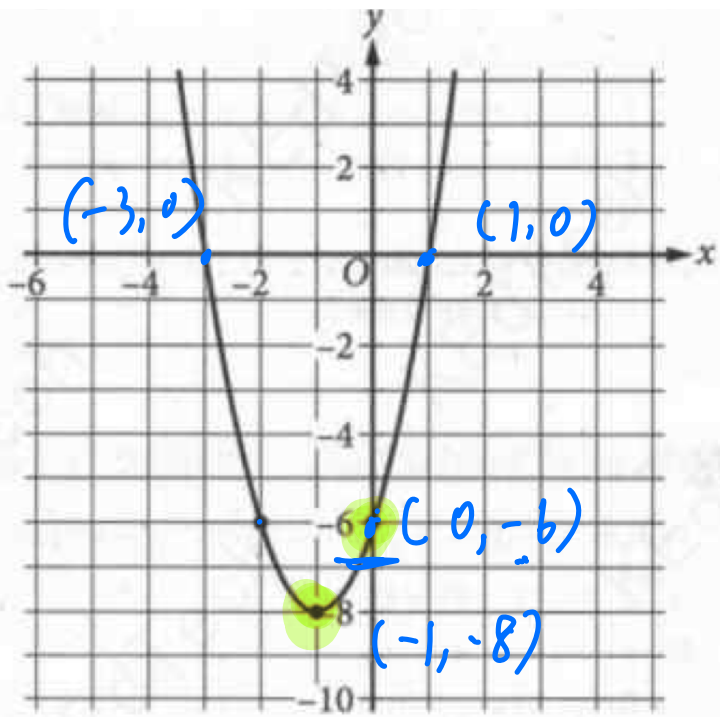
In the given system of equations, p is a constant. If the system has no solution, what is the value of p?

$6+7r=pw$ $\quad 7r=pw-6$

$7r-5w=5w+11$ $\quad 7r=10w+11$

$pw-6=10w+11$ $\quad p=10$

$pw-6=10w+11$



The graph of $y = 2x^2 + bx + c$ is shown, where b and c are constants. What is the value of bc?

$y=2x^2+bx+c$

$-6=2\cdot0^2+b\cdot0+c$

$c=-6$

$-8=2\cdot(-1)^2+b(-1)-6$

$-8=2\cdot1-b-6$

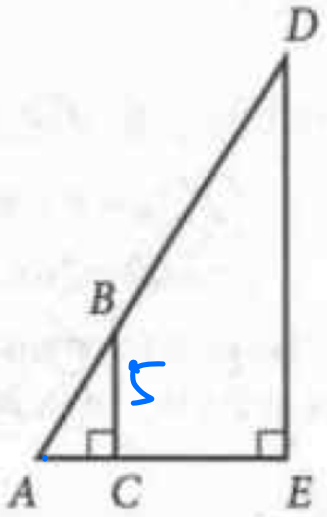
$b=4$ $\quad c=-6$

$4\times(-6)=-24$

$y=2x^2+bx+c$

$=ax^2+bx+c$ $\quad a=2$

$x=-\frac{b}{2a}=-\frac{b}{2\times2}=-1$ $\quad b=4$



Note: Figure not drawn to scale.

In the figure shown, $AB = \sqrt{34}$ units, $AC = 3$ units, and $CE = 21$ units. What is the area, in square units, of triangle ADE?

$AE = 24$

$DE = 40$

$\frac{1}{2} \times 24 \times 40 = 480$

$R_t ABC \sim R_t ADE$

$\frac{AC}{AE} = \frac{BC}{DE}$ $\quad AC = 3 \quad CE = 21$

$AE = 24$

$\frac{AC}{AE} = \frac{3}{24} = \frac{1}{8}$ $\quad AB = \sqrt{34}$

$AB^2 = AC^2 + BC^2 \quad (\sqrt{34})^2 = 3^2 + BC^2$

$BC^2 = (\sqrt{34})^2 - 3^2 = 34 - 9 = 25 = 5^2$

$\frac{BC}{DE} = \frac{5}{DE} = \frac{1}{8}$ $\quad DE = 40$

19 Circles

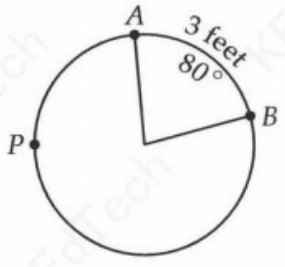
First, the basics. Given a circle with a radius of length r,

- the diameter of the circle is $2r$
- the area of the circle is πr^2
- the circumference of the circle is $2\pi r$

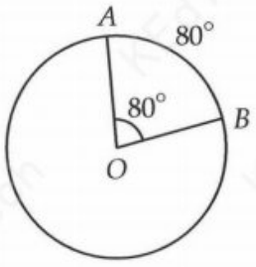
Arc Length vs. Arc Measure

An arc is a portion of a circle. Every arc has an arc length and an arc measure. Arc length is the actual distance along the circle from one point to another (e.g. 3 feet). Arc measure is the number of degrees of rotation along the circle from one point to another (e.g. 80°). A full rotation is 360°.

Minor arcs, such as arc AB below, have a measure of less than 180° and are named using two letters. Major arcs, such as arc APB below, are named using three letters.



The measure of an arc is equal to the measure of the central angle that intercepts it. In the circle below, arc AB has the same measure as central angle AOB.



By definition, a central angle of a circle is an angle whose vertex is at the center of the circle and whose legs are radii. For any given arc, the central angle that intercepts it is sometimes referred to as the arc's **associated angle**. In the example above, angle AOB is arc AB's associated angle.

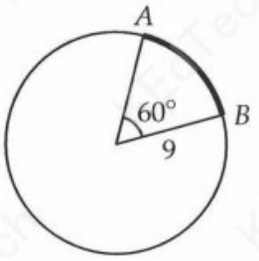
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The length of an arc can be found by using the formula

$$\frac{\theta}{360} \times 2\pi r$$

where θ is the degree measure of the central angle that intercepts the arc and r is the radius. The expression $\frac{\theta}{360}$ represents the fraction of the circle's circumference constituted by the arc, and $2\pi r$ is the circle's circumference.

In the circle below, arc AB constitutes $\frac{60^\circ}{360^\circ} = \frac{1}{6}$ of the circumference. Therefore, its length is $\frac{1}{6} \times 2\pi(9) = 3\pi$ units.



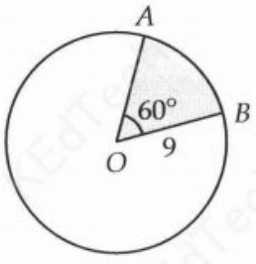
Area of a Sector

A sector is a "slice" of a circle. The area of a sector is given by

$$\frac{\theta}{360} \times \pi r^2$$

where θ is the degree measure of the central angle that encloses the sector and r is the radius. The expression $\frac{\theta}{360}$ represents the fraction of the circle's area constituted by the sector, and πr^2 is the circle's area.

In the circle below, sector OAB constitutes $\frac{60^\circ}{360^\circ} = \frac{1}{6}$ of the area. Therefore, its area is $\frac{1}{6} \times \pi(9)^2 = \frac{27}{2}\pi$ square units.

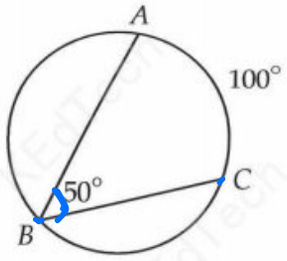


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Angles Inscribed in a Circle

An inscribed angle of a circle is an angle whose vertex lies on the circle and whose legs are chords (line segments whose endpoints both lie on the circle).

The measure of an inscribed angle is half the measure of the arc it intercepts. In the circle below, chords $\overline{AB}$ and $\overline{BC}$ make up inscribed angle ABC, which has half the measure of arc AC.

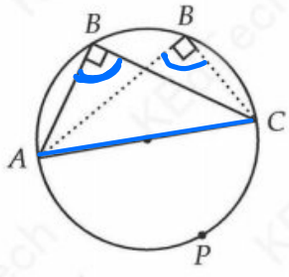


Angles Inscribed in a Semicircle

A semicircle is a half of a circle. We obtain an angle inscribed in a semicircle by designating a point on the circle as the vertex and drawing line segments to the endpoints of a diameter.

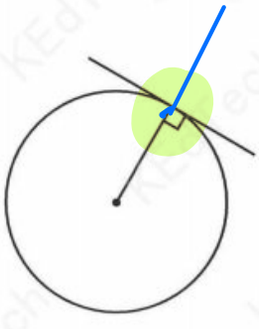
Because angles inscribed in a semicircle always intercept half a circle (180°), they always measure 90° (half of 180°).

In the circle below, $\overline{AC}$ is a diameter and angle ABC is inscribed in semicircle ABC. Because angle ABC intercepts arc APC (180°), angle ABC must be a right angle. Point B could be moved to any distinct location on the circle and angle ABC would still be 90°.



Radius Drawn to a Tangent Line

A line is tangent to a circle if it touches the circle at exactly one point. A radius drawn to that point of tangency is perpendicular to the line:

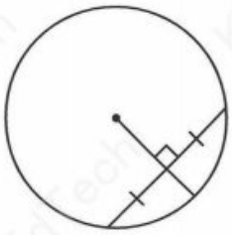


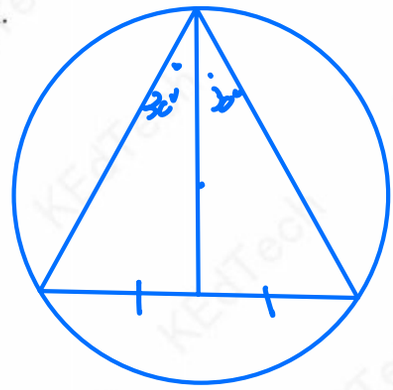
$slope_2 \cdot slope_1 = -1$

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Radius that Bisects a Chord

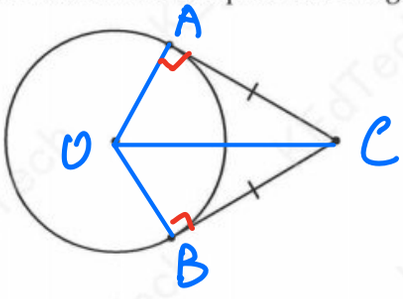
A radius that splits a chord into two equal parts (bisects a chord) is perpendicular to that chord. The converse is also true: a radius that is perpendicular to a chord bisects that chord.





Tangent Segments to a Circle

Tangent segments to a circle drawn from the same external point are congruent.



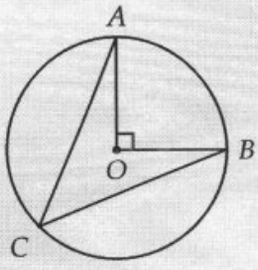
$OC = OC$

$OA = OB$

$OC^2 = OA^2 + AC^2 \Rightarrow AC = BC$

$= OB^2 + BC^2$

EXAMPLE 1:



The circle shown has center O, and points A, B, and C lie on the circle. Angle AOB is a right angle. What is the measure, in degrees, of angle ACB?

A) 15 B) 30 C) 45 D) 60

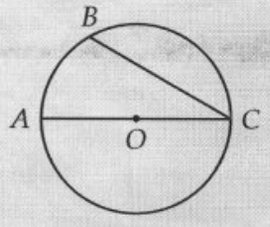
Since central angle AOB ($90°$) intercepts arc AB, arc AB also measures $90°$. Since angle ACB is an inscribed angle that intercepts arc AB, its measure must be half the measure of arc AB: $90° \div 2 = 45°$. Answer $\boxed{(C)}$.

EXAMPLE 2: A circle has radius of length 9 inches. The length of arc AB on this circle is 4π inches. What is the measure, in degrees, of arc AB?

The circumference of the circle is $2\pi r = 2\pi(9) = 18\pi$ inches, which means arc AB constitutes $\frac{4\pi}{18\pi} = \frac{2}{9}$ of the circle. Given that a full circle measures $360°$, arc AB must have a measure of $\frac{2}{9} \times 360° = \boxed{80}$ degrees.

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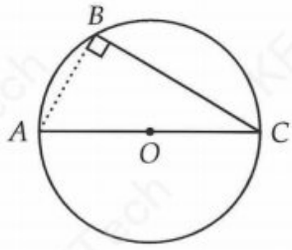
EXAMPLE 3:



In the circle shown, point O is the center, and diameter $\overline{AC}$ has a length of 10 inches. The measure of arc AB is $60°$. What is the length, in inches, of $\overline{BC}$?

A) 6 B) $4\sqrt{3}$ C) 8 D) $5\sqrt{3}$

First, the line that's begging to be drawn is $\overline{AB}$. Since $\overline{AC}$ is a diameter, arc ABC is a semicircle in which angle ABC is inscribed. Angle ABC must therefore be a right angle.



Since arc AB measures $60°$, inscribed angle ACB must measure $60° \div 2 = 30°$. Thus, triangle ABC is a 30–60–90 triangle with hypotenuse $\overline{AC}$ of length 10 inches.

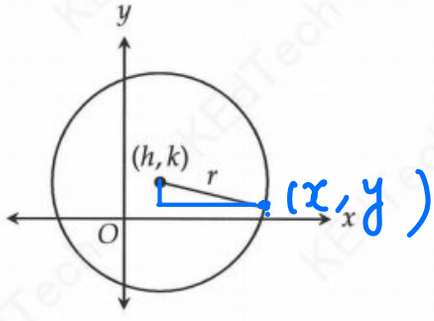
In a 30–60–90 triangle, the shorter leg (e.g. $\overline{AB}$) is half as long as the hypotenuse and the longer leg (e.g. $\overline{BC}$) is $\sqrt{3}$ times longer than the shorter leg. Therefore, the length of side $\overline{AB}$ is $10 \div 2 = 5$ inches, and the length of side $\overline{BC}$ is $5\sqrt{3}$ inches. Answer $\boxed{(D)}$.

Equation of a Circle in the xy-plane

The standard form equation of a circle in the xy-plane is

$$(x-h)^2 + (y-k)^2 = r^2$$

where (h,k) is the center of the circle and r is the length of its radius.



$(x-h)^2 + (y-k)^2 = r^2$

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Let's illustrate with a quick example. Let's say we have the following equation of a circle:

$$(x+6)^2+(y-5)^2=64$$

This equation is in standard form with $h=-6$, $k=5$, and $r=\sqrt{64}=8$. Therefore, the circle is centered at $(-6,5)$ and has a radius of 8.

When finding the center of a circle, the foolproof method I like to use is to ask myself what values make the expressions inside the parentheses equal to zero. *What value of x makes $x+6$ equal to zero? And what value of y makes $y-5$ equal to zero?* Well, $x=-6$ and $y=5$. Therefore, the center is at $(-6,5)$.

EXAMPLE 4: The equation $x^2+y^2=9$ represents a circle in the xy-plane. A new circle is drawn whose center is 8 units down from the center of this circle and whose radius is 7 units longer than the radius of this circle. Which of the following is an equation of this new circle?

A) $x^2+(y-8)^2=16$
B) $x^2+(y+8)^2=16$
C) $x^2+(y-8)^2=100$
D) $x^2+(y+8)^2=100$

The given circle has center $(0,0)$ and a radius of $\sqrt{9}=3$. The new center is 8 units down, so its coordinates must be $(0,0-8)=(0,-8)$. The new radius is 7 units longer, so its length must be $3+7=10$. The equation of the new circle is then

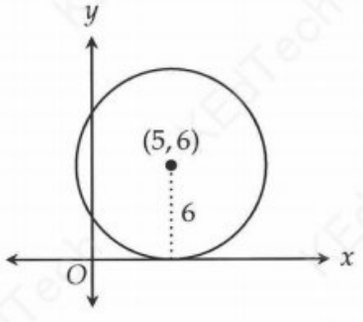
$$(x-0)^2+(y-(-8))^2=10^2$$
$$x^2+(y+8)^2=100$$

Answer $\boxed{(D)}$.

EXAMPLE 5: A circle in the xy-plane has center $(5,6)$ and intersects the x-axis at exactly one point. Which of the following equations represents this circle?

A) $(x-5)^2+(y-6)^2=25$
B) $(x+5)^2+(y+6)^2=25$
C) $(x-5)^2+(y-6)^2=36$
D) $(x+5)^2+(y+6)^2=36$

The circle's center is 5 units to the right of the y-axis and 6 units above the x-axis. For the circle to intersect the x-axis at exactly one point (i.e. be tangent to the x-axis), it must have a radius that is equal to the center's distance from the x-axis. Therefore, the circle must have a radius of 6.



The equation of the circle is then $(x-5)^2+(y-6)^2=36$. Answer $\boxed{(C)}$.

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