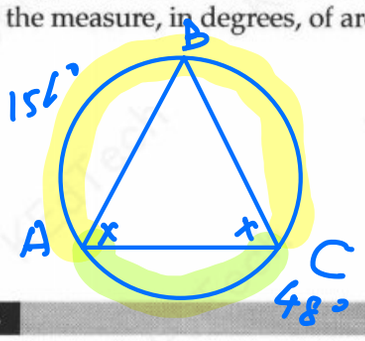
7

Triangle ABC is inscribed in a circle. The measure of angle A is equal to the measure of angle C. The measure of arc AC is 48°. What is the measure, in degrees, of arc AB ?



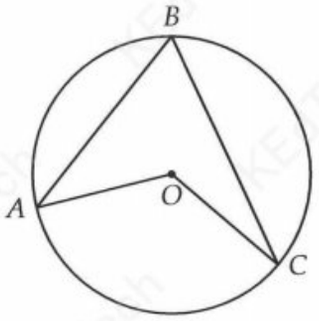
$\angle A = \angle C$

$\overset{\frown}{AC} = 48°$

8

In the xy-plane, circle A is defined by the equation $(x-3)^2+(y-4)^2=16$, and circle B is defined by the equation $(x-3)^2+(y-4)^2=n$, where n is a constant. The diameter of circle B is 48 units longer than the diameter of circle A. What is the value of n ?

9



The circle shown has center O, and points A, B, and C lie on the circle. The measure of angle BAO is 38° and the measure of angle BCO is 25°. What is the measure, in degrees, of arc AC ?

$x^2-18x+81+y^2+24y+144=1$

$x^2+y^2-18x+24y+81+144-1=0$

$a \quad b \quad c$

$a=-18 \quad b=24 \quad c=224$

$a+b+c: -18+24+224=230$

10

Which of the following equations represents a circle in the xy-plane that intersects the line $x=5$ at exactly one point?

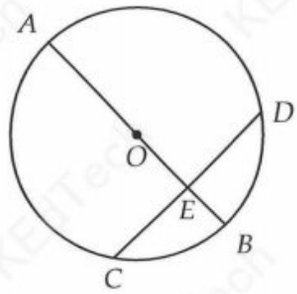
A) $(x-1)^2+(y-7)^2=4$

B) $(x-5)^2+(y-1)^2=4$

C) $(x-7)^2+(y-5)^2=4$

D) $(x-13)^2+(y-13)^2=4$

11



Note: Figure not drawn to scale.

The circle shown has center O and a radius of length 16. Diameter $\overline{AB}$ bisects $\overline{CD}$ at point E, and $CD=28$. If the length of $\overline{OE}$ can be written in the form $2\sqrt{k}$, where k is an integer, what is the value of k ?

12

In the xy-plane, a circle with a radius of 1 has its center at $(9,-12)$. This circle can be represented by the equation $x^2+y^2+ax+by+c=0$, where a, b, and c are constants. What is the value of $a+b+c$?

$(x-h)^2+(y-k)^2=r^2$

$(x-9)^2+(y-(-12))^2=1^2$

$(x-9)^2+(y+12)^2=1$

K Education & Technology

13

A circle in the xy-plane is defined by the equation $(x-h)^2+(y-k)^2=144$, where h and k are constants. The point $(8,27)$ lies on this circle. Which of the following is a possible value for k?

A) -8
B) 12
C) 39
D) 43

14

In the xy-plane, the points $(-4,3)$ and $(8,3)$ are endpoints of a diameter of a circle. An equation of this circle is $(x-a)^2+(y-b)^2=c^2$, where a, b, and c are positive constants. What is the value of $a+b+c$?

11

15

A circle has center A, and points B and C lie on the circle. Line segment BD is tangent to the circle at point B, line segment CD is tangent to the circle at point C, and the length of line segment BD is 63 units. If the circle has a radius of 16 units, what is the area, in square units, of quadrilateral $ABCD$?

1008

16

$$x^2+y^2+7x+y=-\frac{7}{2}$$

The given equation represents a circle in the xy-plane. What is the length of the radius of the circle?

3

17

In the xy-plane, a circle has center D with coordinates $(m,16)$, where m is a constant. Points E and F lie on the circle. Point E lies on the circle and has coordinates $(m+4\sqrt{10},34)$, and point F has coordinates $(n,16)$, where n is a constant greater than m. What is the area, in square units, of triangle DEF?

A) 198
B) $72\sqrt{10}$
C) 242
D) $99\sqrt{10}$

18

A circle in the xy-plane has the equation $(x-2)^2+(y-2)^2=25$. Line m is tangent to this circle at the point $(5,6)$. What is the y-coordinate of the y-intercept of line m?

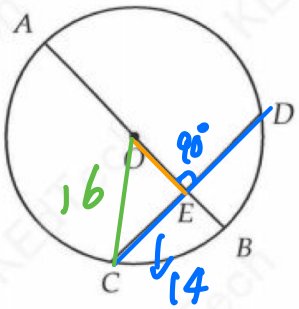
(2, 2)
r = 5
(5,6)
B
O(2,4)
m
OB ⊥ BM
$\frac{6-2}{5-2}=\frac{4}{3}$
$-1=\frac{4}{3}\cdot k_m$ $k=-\frac{3}{4}$
$y-6=-\frac{3}{4}(x-5)$
$y-6=-\frac{3}{4}x+\frac{15}{4}$

K Education & Technology

$$y = -\frac{3}{4}x + \frac{15}{4} + 6$$

$$= \frac{15+24}{4} = \frac{39}{4}$$

11



Note: Figure not drawn to scale.

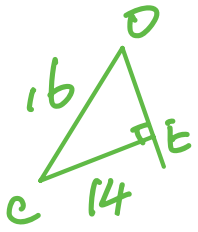
The circle shown has center O and a radius of length 16. Diameter $\overline{AB}$ bisects $\overline{CD}$ at point E, and $CD = 28$. If the length of $\overline{OE}$ can be written in the form $2\sqrt{k}$, where k is an integer, what is the value of k ?

$CE = ED$

$OE \perp CE$

$CE = 14$

$r = 16$



$16^2 - 14^2 = OE^2$

$OE = \sqrt{16^2 - 14^2}$

$2\sqrt{k} = 2\sqrt{15}$

$k = 15$

20 Radians

A radian is simply another unit of angle measure. Just as we have feet and meters for distances and pounds and kilograms for weights, we have both degrees and radians for angles.

$$\pi \text{ radians} = 180^\circ$$

If you've never used radians before, don't be put off by the π. After all, it's just a number. We could've written

$$3.14 \text{ radians} \approx 180^\circ$$

instead, but 3.14 is only an approximation. Values are typically expressed in terms of π when radians are being used.

EXAMPLE 1: An angle has a measure of 45 degrees. What is the measure of this angle, in radians?

A) $\frac{\pi}{6}$ B) $\frac{\pi}{4}$ C) $\frac{\pi}{3}$ D) $\frac{\pi}{2}$

To convert from degrees to radians, we use the conversion factor $\frac{\pi \text{ radians}}{180^\circ}$.

$$45^\circ \times \frac{\pi \text{ radians}}{180^\circ} = \frac{\pi}{4} \text{ radians}$$

Answer $\boxed{(B)}$. Notice that the degree units (represented by the circle symbols) cancel out just as they should in any conversion problem.

EXAMPLE 2: An angle has a measure of $\frac{3\pi}{2}$ radians. What is the measure of the angle, in degrees?

This time, we're going from radians to degrees, so we should flip the conversion factor we used in Example 1:

$$\frac{3\pi}{2} \text{ radians} \times \frac{180^\circ}{\pi \text{ radians}} = \boxed{270}^\circ$$

K Education & Technology

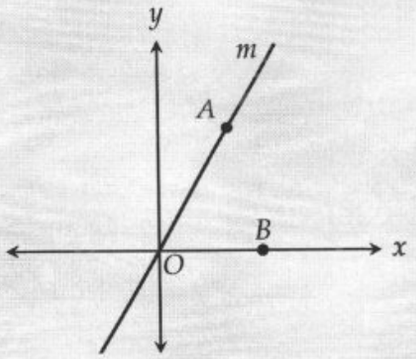
You might be wondering why we even need radians. Why not just stick with degrees? Is this another difference between the U.S. and the rest of the world, like it is with feet and meters? Nope. Some calculations are much simpler when the angles are in radians. For example, the length of an arc or the area of a sector can be found very easily when the central angle is in radians:

$\frac{\theta}{2\pi} = \frac{\text{Arc Length}}{2\pi r}$ $\quad$ Arc Length $= r\theta$ $\quad$ Area of a Sector $= \frac{1}{2}r^2\theta$ $\quad$ $\frac{\theta}{2\pi} = \frac{\text{Area}}{\pi r^2}$

$\theta \cdot r = \text{Arc Length}$ $\quad$ $\frac{\theta \cdot r^2}{2} = \text{Area}$

In these formulas, r is the length of the circle's radius and θ is the radian measure of the central angle. These formulas are nice to know, but you don't need to have them memorized for the SAT.

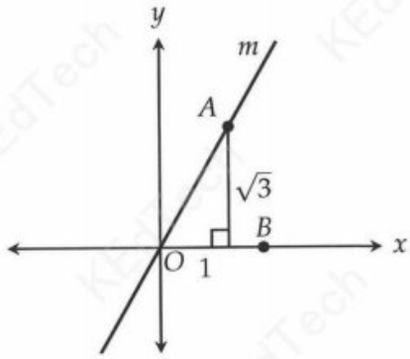
EXAMPLE 3:



In the xy-plane, point A lies on line m and point B lies on the x-axis as shown. If line m passes through the point $(0,0)$ and has a slope of $\sqrt{3}$, what is the measure, in radians, of angle AOB ?

A) $\frac{\pi}{6}$ B) $\frac{\pi}{5}$ C) $\frac{\pi}{4}$ D) $\frac{\pi}{3}$

We can draw a line segment from point A down to the x-axis to make a right triangle. Because the slope of line m is $\sqrt{3}$, the ratio of the height of this triangle to its base is always $\sqrt{3}$ to 1 (rise over run).



In other words, this right triangle has legs whose lengths are in a ratio of $\sqrt{3}$ to 1. It must therefore be a 30–60–90 triangle, in which the angle opposite the $\sqrt{3}$-side is $60°$. The measure of angle AOB is then

$$60° \times \frac{\pi \text{ radians}}{180°} = \frac{\pi}{3} \text{ radians}$$

Answer $\boxed{(D)}$.

K Education & Technology

EXERCISE: Answers for this exercise start on page 392.

1

An angle has a measure of $\frac{17\pi}{12}$ radians. What is the measure of the angle, in degrees?

$180° = \pi$

$\frac{180°}{\pi} \times \frac{17\pi}{12} = \frac{180 \times 17}{12}$

$= 255°$

2

An angle has a measure of 225 degrees. What is the measure of this angle, in radians?

A) $\frac{3\pi}{4}$

B) $\frac{7\pi}{6}$

C) $\frac{5\pi}{4}$

D) $\frac{3\pi}{2}$

$225° \times \frac{\pi}{180°}$

3

The measure of angle A is $\frac{5\pi}{9}$ radians. The measure of angle B is $\frac{7\pi}{36}$ radians less than the measure of angle A. What is the measure of angle B, in degrees?

A) 35

B) 65

C) 100

D) 135

$A = 100°$

$\frac{7\pi}{36} =$

4

The measure of angle P is 162 degrees greater than the measure of angle R. How much greater is the measure of angle P than the measure of angle R, in radians?

A) $\frac{9}{20}\pi$

B) $\frac{9}{10}\pi$

C) $\frac{10}{9}\pi$

D) $\frac{20}{9}\pi$

5

An arc of a circle has a measure of 3.4 radians. What is the measure, in degrees, of this arc, to the nearest degree? (Disregard the degree sign when entering your answer. For example, if your answer is 36°, enter 36)

6

A circle has center O, and points A and B lie on the circle. The length of arc AB is $\frac{1}{5}$ of the circumference of the circle. What is the measure, in radians, of angle AOB ?

A) $\frac{\pi}{10}$

B) $\frac{\pi}{5}$

C) $\frac{2\pi}{5}$

D) $\frac{5\pi}{2}$

K Education & Technology