**7**

The value of $\tan\frac{49\pi}{6}$ can be written as $k\sqrt{3}$, where $k$ is a constant. What is the value of $k$ ?

A) $\frac{1}{3}$

B) $\frac{1}{\sqrt{3}}$

C) 1

D) 3

**8**

Triangle $LMN$ is similar to triangle $QRS$, where $L$ corresponds to $Q$ and angles $M$ and $R$ are right angles. If $\tan(N) = \frac{5}{9}$ and $QR = 4$, what is the length of $\overline{RS}$ ?

**9**

Triangle $JKL$ is similar to triangle $RST$, where angle $J$ corresponds to angle $R$ and angles $K$ and $S$ are right angles. If $\cos(L) = \frac{119}{169}$, what is the value of $\tan(T)$ ?

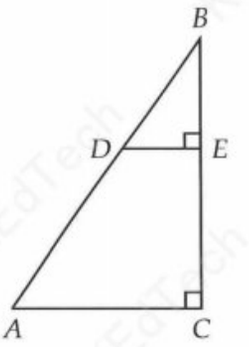
A) $\frac{120}{169}$

B) $\frac{119}{120}$

C) $\frac{120}{119}$

D) $\frac{169}{119}$

**10**




Note: Figure not drawn to scale.

In right triangle $ABC$ shown, $DE = 10$ and $\tan(A) = \frac{5}{4}$. What is the length of $\overline{BE}$ ?

**11**

Triangle $ABC$ is similar to triangle $DEF$, where angle $A$ corresponds to angle $D$ and angles $B$ and $E$ are right angles. If $AB = 2$, $DF = 12\sqrt{2}$, and $\tan(A) = \sqrt{7}$, what is the length of $\overline{DE}$ ?

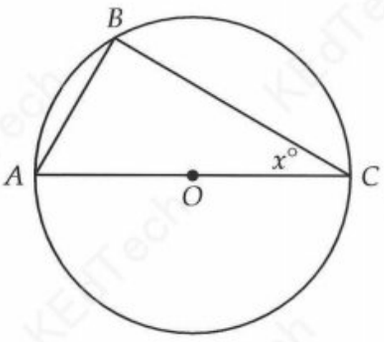


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**12**

In triangle $ABC$, the measure of angle $A$ is equal to the measure of angle $C$. If $\cos A = \frac{3}{8}$ and $AB = 20$, what is the length of $\overline{AC}$?

**13**




The circle shown has center $O$ and diameter $\overline{AC}$. Point $B$ lies on the circle. If $AC = 1$, which of the following represents the area of triangle $ABC$?

A) $\frac{\tan x}{2}$

B) $2\sin x$

C) $\frac{\sin x \cos x}{2}$

D) $\frac{\sin x \tan x}{2}$



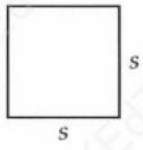
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# 22 Area, Perimeter, & Volume

Since we've already discussed polygons, triangles, and circles in previous chapters, the focus of this chapter will be on geometrical shapes and concepts we haven't covered yet.

## Squares, rectangles, and parallelograms

Square



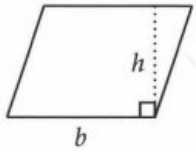

Area $= s^2$
Perimeter $= 4s$

Rectangle




Area $= lw$
Perimeter $= 2l + 2w$

Parallelogram




Area $= bh$

Parallelograms are quadrilaterals in which opposite sides are parallel and equal in length. Rectangles are parallelograms that have four right angles. Squares are rectangles with four sides of equal length.

**EXAMPLE 1:** A square has a perimeter of 76 inches. What is the area of this square, in square inches?

A) 19 B) 361 C) 1,444 D) 5,776

The square must have a side length of $76 \div 4 = 19$ inches. Therefore, its area is $s^2 = 19^2 = 361$ square inches.

Answer $\boxed{(B)}$.



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**EXAMPLE 2:** A rectangle has a width that is 3 inches less than its length. If the area of the rectangle is 108 square inches, what is the perimeter, in inches, of the rectangle?

Let the length of the rectangle be $l$. Its width $w$ is then $l-3$. Since the area of the rectangle is 108 in$^2$,

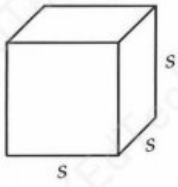
$$\begin{aligned} lw &= 108 \\ l(l-3) &= 108 \\ l^2 - 3l - 108 &= 0 \\ (l-12)(l+9) &= 0 \end{aligned}$$

Since the length of a rectangle has to be positive, $l = 12$ and $w = l - 3 = 12 - 3 = 9$. Finally, the perimeter is $2l + 2w = 2(12) + 2(9) = \boxed{42}$ inches.

## Right prisms and cylinders

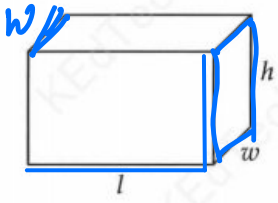
In geometry, a **solid** is a 3-dimensional shape. A **prism** is a solid with flat sides and two identical ends (often called bases). A **right prism** is a prism in which the sides are perpendicular to the bases. In other words, a right prism "stands up straight." The prisms you encounter on the SAT will be right prisms:

Cube




- Volume $= s^3$
- Surface area $= 6s^2$

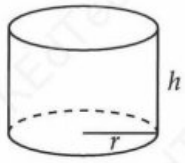
Right Rectangular Prism




Volume $= lwh$

Surface area $= 2lw + 2lh + 2wh$

Right Circular Cylinder




Volume $= \pi r^2 h$

The volume of any right prism can be found using the following formula:

$$\text{Volume} = \text{Area of base} \times \text{Height}$$

This formula explains why the volume of a cube is $s^3$ (base area of $s^2$ and height of $s$), the volume of a right rectangular prism is $lwh$ (base area of $lw$ and height of $h$), and the volume of a right circular cylinder is $\pi r^2 h$ (base area of $\pi r^2$ and height of $h$). Note that a cylinder is technically not a prism by definition but the concept still applies.

The **surface area** of a solid is the total area that the surface of the solid occupies. Since a cube has 6 faces and each face has an area of $s^2$, the surface area of a cube is $6s^2$. A right rectangular prism, on the other hand, has 2 faces each with an area of $lw$, 2 faces each with an area of $lh$, and 2 faces each with an area of $wh$, which together yield a surface area of $2lw + 2lh + 2wh$. The surface area of a right circular cylinder is derived in Example 4.



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**EXAMPLE 3:** A right rectangular prism has a length of 15 inches and a width of 18 inches. If the volume of the prism is 7,020 cubic inches, what is the surface area, in square inches, of this prism?

First, we need to solve for the height $h$. Since the volume of the prism is 7,020 cubic inches,

$$\begin{aligned} lwh &= 7{,}020 \\ (15)(18)h &= 7{,}020 \\ h &= 7{,}020 \div (15 \cdot 18) = 26 \text{ inches} \end{aligned}$$

The surface area of the prism is then $2lw + 2lh + 2wh = 2(15)(18) + 2(15)(26) + 2(18)(26) = \boxed{2256}$ in$^2$.

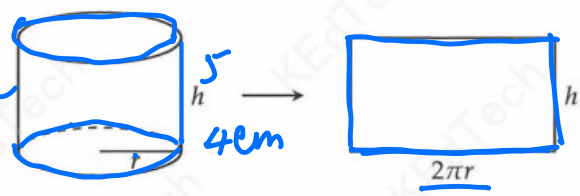
**EXAMPLE 4:** A right circular cylinder has a radius of 4 centimeters (cm) and a height of 5 cm. What is the surface area, in cm$^2$, of this cylinder?

A) $40\pi$ B) $60\pi$ C) $72\pi$ D) $81\pi$

For the sake of learning, let's first derive the formula for the surface area of a cylinder with radius $r$ and height $h$. Then we'll apply the formula to this question.

The surface area of a cylinder consists of the areas of its two bases and the area of the curved surface that goes around the cylinder. Since the two bases are circles and each circle has an area of $\pi r^2$, the two bases contribute a total surface area of $2\pi r^2$. That's the easy part.

To find the area of the curved surface that goes around the cylinder, we have to imagine that the curved surface is unraveled like wrapping paper:




Unraveling the curved surface results in a rectangle with a length equal to the cylinder's base circumference, $2\pi r$, and a width equal to the height of the cylinder, $h$. The area of this rectangle is the area of the curved surface: $lw = 2\pi rh$.

Summing up the areas of the bases and the area of the curved surface, we get the following formula:

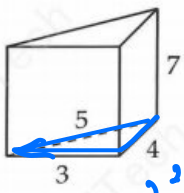
$$\text{Surface area of a cylinder} = 2\pi r^2 + 2\pi rh$$

Applying this formula to the cylinder given in the question, we get a surface area of $2\pi(4)^2 + 2\pi(4)(5) = 32\pi + 40\pi = 72\pi$. Answer $\boxed{(C)}$.



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**EXAMPLE 5:** A triangular prism has a height of 7 inches. The base of the prism is a right triangle with sides of length 3, 4, and 5 inches. What is the volume, in cubic inches, of the prism?



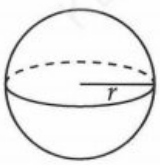

Since the base of the prism is a 3–4–5 right triangle (the 5-inch side being the hypotenuse), the area of the base is $\frac{1}{2}(\text{leg}_1)(\text{leg}_2) = \frac{1}{2}(3)(4) = 6$ square inches. The volume of the prism is then the area of the base multiplied by the height of the prism: $6 \times 7 = \boxed{42}$ cubic inches.

$3^2 + 4^2 = 5^2$

## Spheres, Cones, and Pyramids

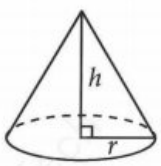
The following solids also appear on the SAT, but not as often as cubes, rectangular prisms, and cylinders. Even though the SAT gives you these formulas at the beginning of each math section, all the volume formulas presented in this chapter are worth memorizing.

Sphere



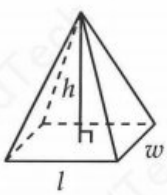

Volume $= \frac{4}{3}\pi r^3$

Cone




Volume $= \frac{1}{3}\pi r^2 h$

Square Pyramid




Volume $= \frac{1}{3}lwh$

## Changes that affect area and volume

How does doubling both the length and the width of a rectangle affect its area? How does increasing the radius of a sphere by 50% affect its volume? The following examples will show you how to tackle these types of questions.

**EXAMPLE 6:** Circle A has a radius that is 30% longer than the radius of circle B. The area of circle A is $k$ percent greater than the area of circle B. What is the value of $k$?

A) 9 B) 30 C) 36 D) 69

$r_A = 1.03 r_B$

$\pi r_A^2 = \pi \cdot (1.02)^2 r_B^2$

$1.69 - 1 = 0.69$

If we let $r_a$ be the radius of circle A and $r_b$ be the radius of circle B, then $r_a = 1.30 r_b$ (if you don't understand why, see the Percent chapter). The area of circle A can then be expressed as

$$\pi r_a^2 = \pi(1.30 r_b)^2 = \pi \cdot (1.30)^2 \cdot r_b^2 = (1.30)^2 \pi r_b^2 = 1.69 \pi r_b^2$$

Note the use of parentheses when we substitute $1.30 r_b$ for $r_a$ in the first step. These parentheses are very important. In the following steps, we just rearrange the terms until there is a single coefficient, 1.69, in front of $\pi r_b^2$, the area of circle B.

This coefficient indicates that the area of circle A is 1.69 times the area of circle B. The area of circle A is therefore 69 percent greater than the area of circle B. Answer $\boxed{(D)}$. Here's another way to interpret this result—increasing the radius of a circle by 30% will increase its area by 69%.



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**EXAMPLE 7:** Sphere P has a radius that is 5 times the radius of sphere Q. The volume of sphere P is how many times the volume of sphere Q?

If we let $r_p$ be the radius of sphere P and $r_q$ be the radius of sphere Q, then $r_p = 5r_q$. The volume of sphere P is then

$$\frac{4}{3}\pi r_p^3 = \frac{4}{3}\pi(5r_q)^3 = \frac{4}{3}\pi \cdot (5)^3 \cdot r_q^3 = (5)^3\left(\frac{4}{3}\pi r_q^3\right) = 125\left(\frac{4}{3}\pi r_q^3\right) = \boxed{125} \times \text{the volume of sphere Q}$$

Notice that 125 is just $5^3$. The factor of 5 gets cubed because of the $r^3$ in the volume formula. Using this "shortcut," we can easily determine what happens in other scenarios. For example, increasing the radius of a sphere by a factor of 6 would result in a volume that is $6^3 = 216$ times larger. By reverse logic, a sphere whose volume is 216 times the volume of a smaller sphere must have a radius that is $\sqrt[3]{216} = 6$ times longer than that of the smaller sphere.

**EXAMPLE 8:** Cones A and B are right circular cones. Cone A has a radius that is $\frac{1}{2}$ the radius of cone B and a height that is 3 times the height of cone B. Cone B has a volume of 720 cubic inches. What is the volume of cone A, in cubic inches?

Let $r_a$ and $h_a$ be the radius and height of cone A, and let $r_b$ and $h_b$ be the radius and height of cone B. Based on the information given, $r_a = \frac{1}{2}r_b$ and $h_a = 3h_b$. The volume of cone A can then be expressed as

$$\frac{1}{3}\pi r_a^2 h_a = \frac{1}{3}\pi\left(\frac{1}{2}r_b\right)^2(3h_b) = \frac{1}{3}\pi \cdot \left(\frac{1}{2}\right)^2 \cdot r_b^2 \cdot 3 \cdot h_b = \left(\frac{1}{2}\right)^2 \cdot 3 \cdot \left(\frac{1}{3}\pi r_b^2 h_b\right) = \frac{3}{4}\left(\frac{1}{3}\pi r_b^2 h_b\right)$$

Therefore, the volume of cone A is $\frac{3}{4}$ the volume of cone B: $720 \times \frac{3}{4} = \boxed{540}$ in$^3$.

## Similar polygons and solids

Two polygons are similar if their corresponding angles are congruent and their corresponding sides are in the same proportion. Two solids are similar if they are of the same type and their corresponding dimensions are in the same proportion. Similar polygons and similar solids are essentially different size versions of the same shape/solid.



Similar Trapezoids



Similar Rectangular Prisms

If two polygons are similar, then the ratio of their areas is equal to the **square** of the ratio of their corresponding side lengths. If two solids are similar, then the ratio of their volumes is equal to the **cube** of the ratio of their corresponding dimensions.

So let's say that triangles A and B are similar. If triangle A has side lengths that are 3 times the corresponding side lengths of triangle B (ratio of 3:1), then the area of triangle A is 9 times the area of triangle B (ratio of $3^2{:}1^2 = 9{:}1$).



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