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Rates, Conversions, & Ratios

The best way to approach most questions involving rates is to use conversion factors (if you've gone through high school chemistry, you should know what I'm talking about). Conversion factors are multipliers that help you go from one set of units to another. For example, $\frac{12 \text{ months}}{1 \text{ year}}$ is a conversion factor that converts years into months. Not only can they represent standard unit conversions (e.g. $\frac{60 \text{ minutes}}{1 \text{ hour}}$), they can also represent information provided by the question (e.g. $\frac{1 \text{ ticket}}{50 \text{ dollars}}$). When using conversion factors to solve a problem, you must set them up in the right sequence and with the appropriate numerators and denominators.

EXAMPLE 1: A bicycle manufacturer can produce 20 bicycles per hour. How many hours would it take the manufacturer to produce 320 bicycles?

Intuitive Approach: We divide the total by the rate to get $320 \div 20 = \boxed{16}$ hours. Easy.

Conversion Factor Approach: The given rate of 20 bicycles per hour can be used as a conversion factor: either $\frac{20 \text{ bicycles}}{1 \text{ hour}}$ or its reciprocal, $\frac{1 \text{ hour}}{20 \text{ bicycles}}$. In this case, we want to use the conversion factor $\frac{1 \text{ hour}}{20 \text{ bicycles}}$ so that when we multiply by the given 320 bicycles, the "bicycles" unit cancels out and the "hours" unit is what remains:

$$320 \cancel{\text{ bicycles}} \times \frac{1 \text{ hour}}{20 \cancel{\text{ bicycles}}} = \frac{320 \text{ hours}}{20} = \boxed{16} \text{ hours}$$

Having the correct unit remaining at the end confirms that we used the correct conversion factor. If the question had asked us to convert a given number of hours into the number of bicycles that can be made, we would've used the other conversion factor, $\frac{20 \text{ bicycles}}{1 \text{ hour}}$.

Although the conversion factor approach seems tedious, I recommend using this approach by default to keep your work organized and to avoid silly mistakes, especially for the harder problems. Ultimately, you are using conversion factors in some form or fashion whether you reason out the conversions in your head or write them out explicitly.

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EXAMPLE 2: A box at the supermarket can hold 6 oranges each. Each orange costs 20 cents. Given that the supermarket has a budget of \$540 to stock oranges, how many boxes will the supermarket be able to fill?

Intuitive Approach: If each orange is 20 cents, then a dollar would be enough for 5 oranges. Five hundred forty dollars would then be enough for $540 \times 5 = 2700$ oranges, which would fill $2700 \div 6 = \boxed{450}$ boxes.

Conversion Factor Approach: Typically, our setup will begin with a starting constraint given by the question (in this case, 540 dollars) or a target that needs to be reached (e.g. 320 bicycles in Example 1).

$$540 \text{ dollars} \times \frac{100 \text{ cents}}{1 \text{ dollar}} \times \frac{1 \text{ orange}}{20 \text{ cents}} \times \frac{1 \text{ box}}{6 \text{ oranges}} = \frac{540 \times 100 \text{ boxes}}{20 \times 6} = \boxed{450} \text{ boxes}$$

Notice how the units cancel as we go along. If the units are canceling, chances are we're doing things right. And since the answer should be in boxes, the "box(es)" unit that remains confirms that we've indeed done things correctly.

EXAMPLE 3: A car can travel 1 mile in 1 minute and 15 seconds. At this rate, how many miles can the car travel in 1 hour?

$$1 \text{ hour} \times \frac{60 \text{ minutes}}{1 \text{ hour}} \times \frac{60 \text{ seconds}}{1 \text{ minute}} \times \frac{1 \text{ mile}}{75 \text{ seconds}} = \frac{60 \times 60 \text{ miles}}{75} = \boxed{48} \text{ miles}$$

EXAMPLE 4: Tom drives at an average speed of 50 miles per hour. Which expression represents the number of hours it will take Tom to drive $5x$ miles at this speed?

A) $\frac{x}{10}$ B) $\frac{10}{x}$ C) $10x$ D) $250x$

$$5x \text{ miles} \times \frac{1 \text{ hour}}{50 \text{ miles}} = \frac{5x \text{ hours}}{50} = \frac{x}{10} \text{ hours}$$

Answer $\boxed{(A)}$.

EXAMPLE 5: To prepare for class, Mr. Chu has to print a number of booklets with p pages per booklet. If every 5 pages cost c cents to print and he spent a total of d dollars, how many booklets did Mr. Chu print in terms of p, c, and d ?

A) $\frac{cp}{500d}$ B) $\frac{100d}{cp}$ C) $\frac{500d}{cp}$ D) $\frac{5d}{cp}$

$$d \text{ dollars} \times \frac{100 \text{ cents}}{1 \text{ dollar}} \times \frac{5 \text{ pages}}{c \text{ cents}} \times \frac{1 \text{ booklet}}{p \text{ pages}} = \frac{d(100)(5)}{cp} \text{ booklets} = \frac{500d}{cp} \text{ booklets}$$

Answer $\boxed{(C)}$.

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EXAMPLE 6: A certain beach has an area of 87,412 square feet. What is the area, in square meters, of this beach? (Use 1 meter = 3.28 feet.)

A) 2,477.13 B) 8,125 C) 26,650 D) 286,711.36

To convert from one square unit to another square unit using a conversion factor that isn't in square units, we have to use the same conversion factor twice to ensure the old units are adequately cancelled out and the new unit is correctly squared.

$$87{,}412\ \cancel{\text{feet}}^2 \times \frac{1\ \text{meter}}{3.28\ \cancel{\text{feet}}} \times \frac{1\ \text{meter}}{3.28\ \cancel{\text{feet}}} = \frac{87{,}412\ \text{meters}^2}{(3.28)(3.28)} = 8{,}125\ \text{meters}^2$$

Answer $\boxed{(B)}$.

EXAMPLE 7: The speed of an object is 210 inches per second. What is the speed, in feet per minute? (1 foot = 12 inches)

A) 17.5 B) 42 C) 1,050 D) 12,600

When converting speeds, we have to watch out for two units: a unit of length and a unit of time. In this example, our conversion factors need to be set up in such a way that we get $\frac{\text{feet}}{\text{minute}}$ at the end.

$$\frac{210\ \cancel{\text{inches}}}{1\ \cancel{\text{second}}} \times \frac{1\ \text{foot}}{12\ \cancel{\text{inches}}} \times \frac{60\ \cancel{\text{seconds}}}{1\ \text{minute}} = \frac{210 \times 60}{12}\ \frac{\text{feet}}{\text{minute}} = 1{,}050\ \frac{\text{feet}}{\text{minute}}$$

Answer $\boxed{(C)}$.

So far, the examples in this chapter have all dealt with conversion factors. Let's do a simple example where we have to find a rate rather than use one to perform a conversion. To be more specific, we'll sometimes be asked to find the **average rate of change**, which, in most situations, is the change in the value of a quantity divided by the elapsed time.

EXAMPLE 8: A rocket has 360 gallons of fuel left after 2 hours of flight, and only 100 gallons after 6 hours of flight. On average, it burns n gallons of fuel for every hour of flight, where n is a constant. What is the value of n ?

In $6 - 2 = 4$ hours of flight, the rocket burned $360 - 100 = 260$ gallons of fuel. Therefore, the rocket, on average, burns $\frac{260}{4} = \boxed{65}$ gallons of fuel every hour.

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Ratios

Let's say that the ratio of cars to trucks in a parking lot is 5:2 (5 to 2). Because ratios can be written as fractions, this ratio is equivalent to $\frac{5}{2}$. A ratio of 5:2 is also equivalent to a ratio of 10:4, since the latter reduces to the former.

In this context, a ratio of 5:2 means that for every 5 cars, there are 2 trucks. And assuming that there are only cars and trucks parked in the lot, this ratio also means that there are 5 cars for every 7 vehicles. By the same token, there are 2 trucks for every 7 vehicles.

EXAMPLE 9: Minyoung bought croissants and bagels for a breakfast event. The ratio of the number of croissants she bought to the number of bagels she bought was 3 to 4. If Minyoung bought 72 bagels, how many croissants did she buy?

According to the given ratio, Minyoung bought 3 croissants for every 4 bagels. Since she bought 72 bagels, she must have bought

$$\frac{3 \text{ croissants}}{4 \text{ bagels}} \times 72 \text{ bagels} = \boxed{54} \text{ croissants}$$

EXAMPLE 10: Arfand is following a recipe for a seasoning blend that requires sea salt, black pepper, and paprika. According to the recipe, the ratio of grams of sea salt to grams of black pepper should be 1:2, and the ratio of grams of black pepper to grams of paprika should be 4:3. How many grams of paprika should Arfand use to make 108 grams of the seasoning blend?

Since black pepper is involved in both the given ratios, we can use it to establish a "common denominator" for comparison. First, multiply the sea salt to black pepper ratio by 2 to get 2:4. Why multiply by 2? Because now the "4" in this sea salt to black pepper ratio lines up with the "4" in the black pepper to paprika ratio. Now that we have this common number in both ratios, it's easy to see that the ratio between the three ingredients is 2:4:3 (sea salt to black pepper to paprika).

According to this ratio, 3 grams of paprika should be used for every $2+4+3=9$ grams of the blend. Therefore, Arfand should use $\frac{3}{9} \times 108 = \boxed{36}$ grams of paprika.

EXAMPLE 11: If the ratio of $y : 2.4$ is equivalent to $2.7 : 3.6$, what is the value of y?

A) $\frac{3}{2}$ B) $\frac{4}{3}$ C) $\frac{7}{3}$ D) $\frac{9}{5}$

Because ratios are just fractions, we can set up the following equation:

$$\frac{y}{2.4} = \frac{2.7}{3.6}$$
$$y = \frac{2.7}{3.6}(2.4) = 1.8 = \frac{9}{5}$$

Answer $\boxed{(D)}$.

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EXERCISE 1: Answers for this exercise start on page 348.

1

Idina can type 90 words in 2.5 minutes. How many words can she type in 12 minutes?

2

The ratio of markers to pencils in a drawer is 8 to 3. If there are 48 pencils in the drawer, how many markers are there?

3

Emma is given a total of \$270 to buy blankets and pillows. She buys 5 blankets for \$12 each. If each pillow costs \$15 each, how many pillows can she buy?

4

The ratio a to b is equivalent to the ratio 36 to z. When $a = 3$, what is the value of b in terms of z ?

A) $\frac{z}{3}$

B) $12z$

C) $\frac{z}{12}$

D) $108z$

5

A distance of 9 furlongs is equivalent to how many inches? (1 furlong = 220 yards and 1 yard = 36 inches)

6

The ratio of blueberries to grapes in a fruit salad is 5 to 12. If there are x blueberries in the fruit salad, which expression represents the number of grapes in the fruit salad?

A) $\frac{5}{12}x$

B) $\frac{12}{5}x$

C) $\frac{12}{17}x$

D) $60x$

7

An empty pool can be filled in 5 hours if water is pumped in at 300 gallons an hour. How many hours would it take to fill the pool if water is pumped in at 500 gallons an hour?

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Which of the following speeds is equivalent to 990 miles per hour? (1 mile = 1,760 yards)

A) 33.75 yards per second
B) 484 yards per second
C) 2,025 yards per second
D) 29,040 yards per second

9

If a apples cost d dollars, which of the following expressions gives the cost of 20 apples, in dollars?

A) $\frac{20a}{d}$
B) $\frac{20d}{a}$
C) $\frac{a}{20d}$
D) $\frac{20}{ad}$

10

At a food stand, the price of nachos is d dollars. The price of french fries is $0.3d$ dollars. What is the ratio of the price of nachos to the price of french fries?

A) 1 to 3
B) 3 to 1
C) 7 to 3
D) 10 to 3

11

Henry drives 150 miles at 30 miles per hour and then another 200 miles at 50 miles per hour. What was his average speed, in miles per hour, for the entire journey, to the nearest hundredth?

A) 38.89
B) 40.00
C) 42.33
D) 43.58

12

The ratio of milk to flour in a mixture is 16 to 20. If 10 cups of milk are added to the mixture, how many cups of flour must be added to the mixture to maintain this ratio?

13

The area of Wompatuck State Park in Massachusetts is 5.5 square miles. Which of the following is closest to the area of this park, in square kilometers? (Use 1 kilometer = 0.62 mile.)

A) 2.1
B) 3.4
C) 8.9
D) 14.3

14

There are a total of 270 coupons in an advertising brochure. The ratio of expired coupons to unexpired coupons is 5 to 1. How many unexpired coupons are there in the advertising brochure?

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At the start of dinner service, there were 30 diners seated inside a restaurant. At 45 minutes into dinner service, there were 120 diners seated inside. If the number of diners seated inside the restaurant increased at a constant rate after the start of dinner service, how many diners were seated inside at 24 minutes into dinner service?

A) 42
B) 48
C) 72
D) 78

16

A flask contains an acidic solution with a concentration of 7.1×10^{15} hydrogen ions per milliliter. If 4.8×10^{23} hydrogen ions have a total mass of 0.8 grams, which of the following is closest to the concentration, in grams per liter, of the acidic solution?

A) 1.2×10^{-5}
B) 1.2×10^{-8}
C) 1.5×10^{-5}
D) 1.5×10^{-8}

17

An island in the Galápagos has an estimated population of 740 sea lions and an area of 4,200 hectares. Which of the following is closest to the estimated population density, in sea lions per square mile, of the island? (Use 1 square mile = 259 hectares.)

A) 0.0007
B) 0.02
C) 45.6
D) 12,000

18

Jared and Robert are accountants who are tasked with reviewing financial reports. It takes Jared 15 hours, working at a constant rate, to review a report containing 240 pages of financial statements. If Robert works at twice Jared's rate, how many minutes would it take Robert to review a report containing 120 pages of financial statements?

A) 225
B) 345
C) 450
D) 900

19

During a race on a circular racetrack, a racecar burns fuel at a constant rate. After lap 4, the racecar has 22 gallons left in its tank. After lap 7, the racecar has 18 gallons left in its tank. Assuming the racecar does not refuel, after which lap will the racecar have 6 gallons left in its tank?

A) Lap 13
B) Lap 15
C) Lap 16
D) Lap 19

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