11 Word Problems

This chapter will focus on questions that require you to construct an equation or expression from a given situation. You will develop an instinct for translating words into math, setting the right variables, and finally solving for the answer (if necessary).

EXAMPLE 1: A retail store has monthly fixed costs of \$3,000 and monthly salary costs of \$2,500 for each employee. If the store maintains a total of x employees during a certain year, which of the following equations represents the total cost c, in dollars, of running the store for that year?

A) $c = 3{,}000 + 2{,}500x$
B) $c = 12(3{,}000 + 2{,}500x)$
C) $c = 12(3{,}000) + 2{,}500x$
D) $c = 3{,}000 + 12(2{,}500x)$

The monthly cost of running the store is $3{,}000 + 2{,}500x$. To get the cost for an entire year, we multiply by 12 months: $c = 12(3{,}000 + 2{,}500x)$. Answer $\boxed{(B)}$.

EXAMPLE 2: Karter traveled a total of 20 miles by jogging at a speed of 4 miles per hour and biking at a speed of b miles per hour. He spent one more hour jogging than he did biking. Which equation represents this situation, where x represents the number of hours Karter spent jogging?

A) $4x + b(x+1) = 20$ B) $4x + b(x-1) = 20$ C) $\frac{x}{4} + \frac{x+1}{b} = 20$ D) $\frac{x}{4} + \frac{x-1}{b} = 20$

Since Karter spent x hours jogging, he must have spent $x - 1$ hours biking. By jogging at 4 miles per hour for x hours, he traveled $4x$ miles, and by biking at b miles per hour for $x - 1$ hours, he traveled $b(x-1)$ miles. Since he traveled a total of 20 miles, $4x + b(x-1) = 20$. Answer $\boxed{(B)}$.

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EXAMPLE 3: The sum of three consecutive integers is 72. What is the largest of these three integers?

The most important technique in solving word problems is to let a variable be one of the things you don't know. In this problem, we don't know any of the three integers, so we let the smallest one be x. It doesn't matter which number we set as x, as long as we're consistent throughout the problem.

So if x is the smallest, then our consecutive integers are

$$x, x+1, x+2$$

Because they sum to 72, we can set up an equation:

$$\begin{aligned} x+(x+1)+(x+2) &= 72 \\ 3x+3 &= 72 \\ 3x &= 69 \\ x &= 23 \end{aligned}$$

Because x is the smallest, our three consecutive integers must then be 23, 24, and $\boxed{25}$ (the largest).

But what would the solution have looked like if we had let x be the largest integer? Our three integers would've been

$$x-2, x-1, x$$

And our equation would've been

$$\begin{aligned} (x-2)+(x-1)+x &= 72 \\ 3x-3 &= 72 \\ 3x &= 75 \\ x &= 25 \end{aligned}$$

And because x was set to be the largest of the three integers in this scenario, we're already at the answer!

The lesson here is that you should think about which unknown you want to set as the variable. Often times, that unknown will be what the question is asking for. Other times, it will be an unknown you specifically choose to make the problem easier to set up and solve. And sometimes, as was the case in this example, it doesn't matter which unknown you pick; you'll end up with the same answer with the same amount of effort.

EXAMPLE 4: One number is 3 times another number. If they sum to 44, what is the larger of the two numbers?

In this problem, we want to set x to be the smaller of the two numbers. That way, the two numbers can be expressed as x and $3x$. If we let x be the larger of the two, we would have to work with x and $\frac{x}{3}$, and fractions are yucky. Setting up our equation, we get

$$\begin{aligned} x+3x &= 44 \\ 4x &= 44 \\ x &= 11 \end{aligned}$$

Be careful—we're not done yet! The question asks for the larger of the two, so we have to multiply x by 3 to get $\boxed{33}$.

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EXAMPLE 5: Of the children enrolled at a daycare program, 70% attend full-time and the rest attend part-time. If the number of children who attend full-time is 150 more than the number of children who attend part-time, what is the total number of children enrolled at the daycare program?

Since 70% attend full-time, 30% must attend part-time. Now let x be the total number of children enrolled at the program. Then $0.70x$ attend full-time and $0.30x$ attend part-time. Setting up an equation, we get

$$\begin{aligned} 0.70x - 0.30x &= 150 \\ 0.40x &= 150 \\ x = \frac{150}{0.40} &= \boxed{375} \end{aligned}$$

EXAMPLE 6: Albert is 7 years older than Henry. In 5 years, Albert will be twice as old as Henry. How many years old is Albert now?

Let x be Albert's age now. We could've assigned x to be Henry's age, but as we mentioned earlier, assigning the variable to be what the question is asking for is typically the fastest route. Now at this point, some of you might be thinking of assigning another variable to Henry's age. While that would certainly work, it would only add more steps to the solution. Try to stick to one variable unless the question clearly calls for more.

If Albert is x years old now, then Henry must be $x - 7$ years old.
Five years from now, Albert will be $x + 5$ and Henry will be $x - 7 + 5 = x - 2$ years old.

$$\begin{aligned} x + 5 &= 2(x - 2) \\ x + 5 &= 2x - 4 \\ x &= \boxed{9} \end{aligned}$$

EXAMPLE 7: After each of her workouts, Iris consumes 8 ounces of a particular drink that is sold in 16-ounce bottles. Each bottle contains 20 grams of protein. Iris would like to increase her protein intake by 3 grams after each workout. Which equation can Iris use to determine how many additional ounces, x, of the drink she should consume after each workout?

A) $\frac{16}{20}x = 3$ B) $\frac{16}{20}x = 13$ C) $\frac{20}{16}x = 3$ D) $\frac{20}{16}x = 13$

There are 20 grams of protein in every 16 ounces. Therefore, in x additional ounces of the drink, there are $\frac{20}{16}x$ grams of protein. This expression needs to equal 3 since that's the amount of protein Iris would like to increase her intake by. Answer $\boxed{(C)}$.

We can also approach this question starting from the 3 additional grams of protein. There are again 20 grams of protein in every 16 ounces of the drink, so to get those 3 additional grams of protein, Iris would need to consume $\frac{16}{20}(3)$ additional ounces. So, $x = \frac{16}{20}(3)$, which is equivalent to the equation $\frac{20}{16}x = 3$.

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EXAMPLE 8: Jake can run 60 yards per minute. Amy can run 120 yards per minute for the first 10 minutes but then slows down to 20 yards per minute thereafter. If they start running at the same time, after how many minutes t will both Jake and Amy have run the same distance, assuming $t > 10$?

The problem already gives us a variable t to work with. We want to equate Jake's distance run with Amy's.

Jake's distance: $60t$
Amy's distance: $120(10) + 20(t - 10)$

$$\begin{aligned} 60t &= 120(10) + 20(t - 10) \\ 60t &= 1{,}200 + 20t - 200 \\ 40t &= 1{,}000 \\ t &= 25 \end{aligned}$$

After $\boxed{25}$ minutes, they will have run the same distance.

EXAMPLE 9: A tank contains 30 liters of a 10% acetone solution. How many liters of a 50% acetone solution must be added to the tank to obtain a 20% acetone solution?

Wrong Solution: The tank currently contains $30(0.10) = 3$ liters of acetone. To obtain a 20% acetone solution, there would need to be $30(0.20) = 6$ liters of acetone. Therefore, $6 - 3 = 3$ liters of acetone need to be added. Given that we have a 50% acetone solution to work with, we would need to add $3(2) = \boxed{6}$ liters of this solution to contribute the necessary 3 liters of acetone.

This solution is wrong and illustrates a common mistake: forgetting that the total amount of solution in the tank changes once we add to it. When we add 6 liters of a 50% solution to the tank, it's not just the amount of acetone that increases; the total amount of solution in the tank also increases. So we can't just take 20% of the original 30 liters and set that as the target because the target shifts once we add more solution.

Correct Solution: The tank currently contains $30(0.10) = 3$ liters of acetone. Let x be the amount of 50% solution that must be added. Adding x liters of the 50% acetone solution will contribute an additional $0.50x$ liters of acetone and increase the total amount of solution in the tank to $30 + x$ liters. Since we want to obtain a 20% solution, we can set up the following equation:

$$\begin{aligned} \frac{\text{Liters of acetone}}{\text{Liters of solution}} &= 0.20 \\ \frac{3 + 0.50x}{30 + x} &= 0.20 \\ 3 + 0.50x &= 0.20(30 + x) \\ 3 + 0.50x &= 6 + 0.20x \\ 0.30x &= 3 \\ x &= \boxed{10} \end{aligned}$$

So to obtain a 20% solution, 10 liters of the 50% solution must be added. This type of word problem with percentages is very common in chemistry and is typically known as a "mixture" problem.

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EXAMPLE 10: When Alex and Barry work separately from each other, Alex can paint a house in 6 days, and Barry can paint a house in 12 days. Assuming that they each work at a constant rate, how many days will it take Alex and Barry to paint a house if they work together?

This is the typical "work-rate" problem that involves two individuals who work at different rates. The general approach is to use the formula $W = rt$, where W is the amount of work done, r is the overall rate at which work is being done, and t is the time spent. The key thing to note is that the overall rate, r, can be found by summing up the individual rates.

Since Alex can paint a house in 6 days, his rate is $\frac{1}{6}$ of a house per day. Since Barry can paint a house in 12 days, his rate is $\frac{1}{12}$ of a house per day.

Working together, they can paint $\frac{1}{6} + \frac{1}{12} = \frac{2}{12} + \frac{1}{12} = \frac{3}{12} = \frac{1}{4}$ of a house per day. Now we can use $W = rt$, where $W = 1$ (i.e. 1 house) and $r = \frac{1}{4}$, to find the time it will take them to paint one house.

$$W = rt$$

$$1 = \frac{1}{4}t$$

$$4 = t$$

Therefore, it will take Alex and Barry $\boxed{4}$ days to paint one house. This answer makes sense because if Alex can finish a house in 6 days by himself, then it should take less than 6 days if Barry is working alongside him.

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EXERCISE 1: Answers for this exercise start on page 351.

1

According to a model, the temperature of an object was 18 degrees Celsius when it was placed in an oven and increased by 2.4 degrees Celsius per minute while it was in the oven. The model is defined by the equation $y = bx + c$, where y is the temperature, in degrees Celsius, of the object x minutes after it was placed in the oven, and b and c are constants. What is the value of c ?

2

If 3 is subtracted from 3 times the number x, the result is 21. What is the result when 8 is added to half of x ?

A) 1
B) 5
C) 8
D) 12

3

A cheese vendor currently has 175 pounds of mozzarella available for sale. If each pound of mozzarella sells for \$8.75, which of the following functions gives the amount of mozzarella M, in pounds, still available for sale after d dollars worth has been sold?

A) $M(d) = 175 - \frac{d}{8.75}$
B) $M(d) = 175 - 8.75d$
C) $M(d) = 175 - \frac{8.75}{d}$
D) $M(d) = 175(8.75) - d$

4

John and David work as home inspectors. For each inspection, John charges \$0.30 per square foot plus a base fee of \$500. David charges \$0.45 per square foot plus a fee of \$25 per bedroom. If they each charge the same amount to inspect a certain 5-bedroom house, what is the size, in square feet, of the house?

5

Marcia baked 240 cookies and distributed a third of them evenly among c classmates. Each classmate received 4 cookies. Which equation represents this situation?

A) $\frac{3c}{240} = 4$
B) $\frac{240c}{3} = 4$
C) $\frac{240}{3c} = 4$
D) $\frac{240(3)}{c} = 4$

6

Point B lies on line segment AC such that the length of AB is 7 more than twice the length of BC. If the length of AC is 55, what is the length of AB ?

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7

A garden has $\frac{3}{5}$ as many shrubs as flowers. If the garden has 180 more flowers than shrubs, how many flowers are in the garden?

8

To commute from his house to his office in the morning, Carl takes a 20-mile route and drives at an average speed of a miles per hour. To commute from his office back to his house in the evening, Carl takes a 24-mile route and drives at an average speed of b miles per hour. Carl's morning commute takes the same amount of time as his evening commute. Which of the following equations represents the information given?

A) $20a + 24b = 44$

B) $\frac{a}{24} = \frac{b}{20}$

C) $\frac{20}{a} = \frac{24}{b}$

D) $\frac{a}{20} + \frac{b}{24} = 1$

9

Each player in a golf league is assigned to one of 38 groups. The number of groups with 4 players is 3 times the number of groups g with 5 players. There are 2 groups with 6 players. Which equation must be true for the value of g ?

A) $3g + 2 = 38$

B) $4g + 5(3g) + 2(6) = 38$

C) $4g + 2 = 38$

D) $4(3g) + 5g + 2(6) = 38$

10

At a store, the price of a tie is k dollars less than three times the price of a shirt. If a shirt costs \$40 and a tie costs \$30, what is the value of k ?

11

A taxicab charges \$4.75 for the first mile traveled and \$3.50 for each additional mile. A particular passenger travels x miles in the taxicab and is charged a total of \$24. Which equation represents this situation?

A) $3.50x - 4.75 = 24$

B) $8.25x - 3.50 = 24$

C) $3.50x + 1.25 = 24$

D) $3.50x + 4.75 = 24$

12

Joey and Matt were tasked with bringing popcorn to a certain event. Joey brought x pounds of popcorn and Matt brought 4 fewer pounds than Joey did. During the event, 60% of the popcorn they had brought was consumed. If Joey and Matt divided the leftover popcorn evenly between them, which of the following represents the number of pounds of leftover popcorn each of them took?

A) $0.2x - 0.8$

B) $0.4x - 0.8$

C) $0.6x - 1.2$

D) $0.8x - 1.6$

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13

Mitch works 42 hours each week clearing land. It takes him 14 hours to clear 1 acre of land. This week, Mitch would like to work additional hours to earn \$200 more than he usually does. If Mitch works at a constant rate and earns \$875 for each acre of land cleared, which of the following equations can he use to solve for h, the additional number of hours he should work this week?

A) $\frac{14}{875}h = 200$

B) $\frac{14}{875}h = 2{,}825$

C) $\frac{875}{14}h = 200$

D) $\frac{875}{14}h = 2{,}825$

14

Joseph can harvest a field of wheat in 12 days. Patrick can harvest the same field twice as fast as Joseph can. If Joseph and Patrick work together at their respective rates to harvest the entire field in t days, which of the following equations represents this situation?

A) $\frac{12}{t} + \frac{6}{t} = 2$

B) $\frac{1}{12}t + \frac{1}{6}t = 1$

C) $\frac{12}{t} + \frac{24}{t} = 2$

D) $\frac{1}{12}t + \frac{1}{24}t = 1$

15

Mark and Kevin own $\frac{1}{4}$ and $\frac{1}{3}$ of the books on a shelf, respectively. Lori owns the remainder of the books. If Kevin owns 9 more books than Mark, how many books does Lori own?

16

A water tank is connected to two pipes, Pipe A and Pipe B. It takes 4 hours to fill the tank when only Pipe A is in use, and it takes 6 hours to fill the tank when only Pipe B is in use. If it takes m minutes to fill the tank when both Pipe A and Pipe B are in use, what is the value of m ?

17

A flask contains 24 milliliters of a 30% acid solution. How many milliliters of an 8% acid solution must be added to the flask to produce a 16% acid solution?

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