EXAMPLE 7: Sphere P has a radius that is 5 times the radius of sphere Q. The volume of sphere P is how many times the volume of sphere Q?

If we let r_p be the radius of sphere P and r_q be the radius of sphere Q, then $r_p = 5r_q$. The volume of sphere P is then

$$\frac{4}{3}\pi r_p^3 = \frac{4}{3}\pi(5r_q)^3 = \frac{4}{3}\pi \cdot (5)^3 \cdot r_q^3 = (5)^3\left(\frac{4}{3}\pi r_q^3\right) = 125\left(\frac{4}{3}\pi r_q^3\right) = \boxed{125} \times \text{the volume of sphere Q}$$

Notice that 125 is just 5^3. The factor of 5 gets cubed because of the r^3 in the volume formula. Using this "shortcut," we can easily determine what happens in other scenarios. For example, increasing the radius of a sphere by a factor of 6 would result in a volume that is $6^3 = 216$ times larger. By reverse logic, a sphere whose volume is 216 times the volume of a smaller sphere must have a radius that is $\sqrt[3]{216} = 6$ times longer than that of the smaller sphere.

EXAMPLE 8: Cones A and B are right circular cones. Cone A has a radius that is $\frac{1}{2}$ the radius of cone B and a height that is 3 times the height of cone B. Cone B has a volume of 720 cubic inches. What is the volume of cone A, in cubic inches?

Let r_a and h_a be the radius and height of cone A, and let r_b and h_b be the radius and height of cone B. Based on the information given, $r_a = \frac{1}{2}r_b$ and $h_a = 3h_b$. The volume of cone A can then be expressed as

$$\frac{1}{3}\pi r_a^2 h_a = \frac{1}{3}\pi\left(\frac{1}{2}r_b\right)^2(3h_b) = \frac{1}{3}\pi \cdot \left(\frac{1}{2}\right)^2 \cdot r_b^2 \cdot 3 \cdot h_b = \left(\frac{1}{2}\right)^2 \cdot 3 \cdot \left(\frac{1}{3}\pi r_b^2 h_b\right) = \frac{3}{4}\left(\frac{1}{3}\pi r_b^2 h_b\right)$$

Therefore, the volume of cone A is $\frac{3}{4}$ the volume of cone B: $720 \times \frac{3}{4} = \boxed{540}$ in^3.

Similar polygons and solids

Two polygons are similar if their corresponding angles are congruent and their corresponding sides are in the same proportion. Two solids are similar if they are of the same type and their corresponding dimensions are in the same proportion. Similar polygons and similar solids are essentially different size versions of the same shape/solid.



Similar Trapezoids Similar Rectangular Prisms

If two polygons are similar, then the ratio of their areas is equal to the **square** of the ratio of their corresponding side lengths. If two solids are similar, then the ratio of their volumes is equal to the **cube** of the ratio of their corresponding dimensions.

So let's say that triangles A and B are similar. If triangle A has side lengths that are 3 times the corresponding side lengths of triangle B (ratio of 3:1), then the area of triangle A is 9 times the area of triangle B (ratio of $3^2{:}1^2 = 9{:}1$).

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EXAMPLE 9: Polygon A is similar to polygon B. Each side of polygon A has a length of 15 units, and each side of polygon B has a length of 3 units. The area of polygon A is how many times the area of polygon B?

Polygon A has sides that are $15 \div 3 = 5$ times as long as the corresponding sides of polygon B. Therefore, the area of polygon A is $5^2 = \boxed{25}$ times the area of polygon B.

EXAMPLE 10: Rectangular prisms A and B are similar. Each edge length of prism B is 12 times the corresponding edge length of prism A. The volume of prism A is 42 cubic inches. What is the volume, in cubic inches, of prism B?

Since the edges of prism B are 12 times as long as the corresponding edges of prism A, the volume of prism B must be $12^3 = 1{,}728$ times the volume of prism A. The volume of prism B is therefore $1{,}728 \times 42 = \boxed{72576}$ cubic inches.

Density

Density is a substance's mass per unit of volume. Memorize the formula:

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}}$$

Denser objects are heavier relative to their size.

EXAMPLE 11:

The volume of a sample of plastic is 1.5 cubic feet, and its mass is 126 pounds. What is the density, in pounds per cubic foot, of this sample?

A) 42 B) 84 C) 124.5 D) 189

$$\text{Density} = \frac{\text{Mass}}{\text{Volume}} = \frac{126 \text{ pounds}}{1.5 \text{ cubic feet}} = 84 \text{ pounds per cubic foot}$$

Answer $\boxed{(B)}$.

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EXERCISE 1: Answers for this exercise start on page 398.

1

The mass of a foam block is 8 kilograms. The volume of the block is 20 cubic meters. What is the density, in kilograms per cubic meter, of the block?

A) 0.4
B) 2.5
C) 12
D) 28

2

A cube has a volume of 42,875 cubic inches. What is the surface area, in square inches, of the cube?

3

A right square pyramid has a height of 33 centimeters (cm) and a square base with a side length of 14 cm. What is the volume, in cm^3, of this pyramid?

4

A cylinder has a volume of 980π cubic feet and a height of 5 feet. What is the base diameter, in feet, of the cylinder?

5

Square A has an area of 64 square centimeters. The perimeter of square B is 4 times the perimeter of square A. What is the side length, in centimeters, of square B?

A) 16
B) 32
C) 64
D) 128

6

Quadrilateral X is similar to quadrilateral Y, where the longest side of quadrilateral X corresponds to the longest side of quadrilateral Y. The longest side of quadrilateral X has a length of 3 inches, and the longest side of quadrilateral Y has a length of 9 inches. The area of quadrilateral Y is how many times the area of quadrilateral X?

A) 3
B) 9
C) 27
D) 36

7

A right rectangular prism has a length of 16 inches, a width of 18 inches, and a height of 20 inches. What is the surface area, in square inches, of this prism?

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8

The perimeter of square A is 144 inches greater than the perimeter of square B. The side length of square A is how many inches greater than the side length of square B?

A) 12
B) 36
C) 72
D) 144



A: 4a
B: 4b
4a - 4b = 144 ⇒ a - b = 144/4 = 36

9

A rectangle has a width of $(x-8)$ units and a length of $(x+5)$ units. If the rectangle has an area of 68 square units, what is the value of x ?

A) 9
B) 12
C) 15
D) 17

10

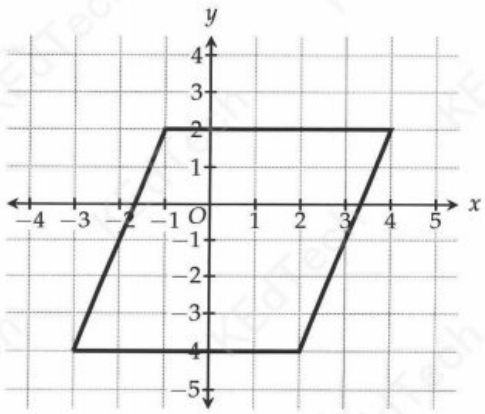
Parallelogram $ABCD$ is similar to parallelogram $PQRS$ such that A, B, C, and D correspond to P, Q, R, and S, respectively. The ratio of the length of side AB to the length of side PQ is 3 to 2. The area of $PQRS$ is 72 square units. What is the area, in square units, of $ABCD$?

A) 32
B) 108
C) 162
D) 324

11

The length of rectangle A is 54 times the length of rectangle B and the width of rectangle A is 45 times the width of rectangle B. The area of rectangle A is n times the area of rectangle B. What is the value of n ?

12



What is the area, in square units, of the parallelogram shown?

13

An object in the shape of a cube has a density of 390 grams per cubic centimeter. The object has a mass of 225 grams. To the nearest hundredth of a centimeter, what is the length of one edge of the object?

A) 0.58
B) 0.76
C) 0.83
D) 1.20

14

The volume of a sphere is equal to the volume of a cube with an edge length of 15 inches. To the nearest tenth of an inch, what is the length of the radius of the sphere?

A) 6.4
B) 9.3
C) 11.3
D) 13.6

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Parallelogram $ABCD$ is similar to parallelogram $PQRS$ such that A, B, C, and D correspond to P, Q, R, and S, respectively. The ratio of the length of side AB to the length of side PQ is 3 to 2. The area of $PQRS$ is 72 square units. What is the area, in square units, of $ABCD$?

A) 32
B) 108
C) 162
D) 324