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Lines & Linear Relationships

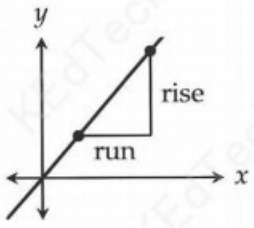
Lines in the xy-plane represent functions of the form $f(x) = mx + b$, which are often referred to as linear functions. Let's dive into some properties you'll need to know.

Given any two points (x_1, y_1) and (x_2, y_2) on a line, the

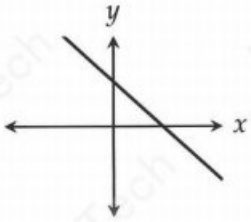
$$\text{slope of the line} = \frac{\text{rise}}{\text{run}} = \frac{y_2 - y_1}{x_2 - x_1}$$

The slope is a measure of the steepness of a line—the larger the slope, the more steep the line is. The rise is the distance between the y coordinates and the run is the distance between the x coordinates. A slope of 2 means that the line goes 2 units up for every 1 unit to the right, or 2 units down for every 1 unit to the left. A slope of $-\frac{2}{3}$ means that the line goes 2 units down for every 3 units to the right, or 2 units up for every 3 units to the left.

Lines with positive slope always go up and to the right as shown below.

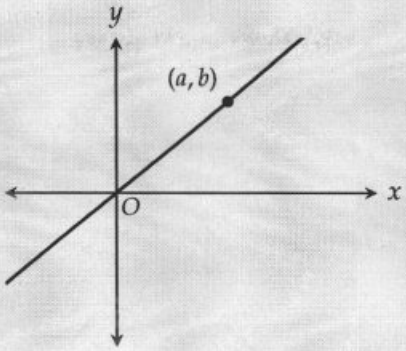


Lines with negative slope go down and to the right:



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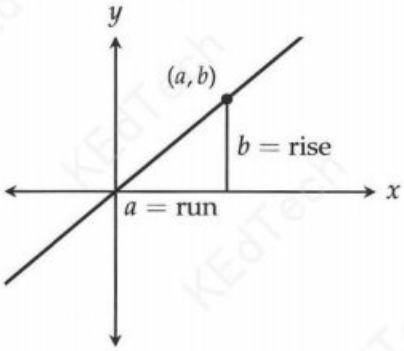
EXAMPLE 1:



The line shown in the xy-plane passes through the point $(0,0)$ and the point (a,b), where $a > b$. Which of the following could be the slope of this line?

A) $-\frac{1}{2}$ B) $\frac{3}{4}$ C) 1 D) $\frac{3}{2}$

First, notice that the slope is positive. The slope, $\frac{\text{rise}}{\text{run}}$, is also equal to $\frac{b}{a}$.



Since $a > b$, $\frac{b}{a}$ is always less than 1. For example, if $a = 5$ and $b = 3$, the slope would be $\frac{3}{5}$. The only choice that's both positive and less than 1 is answer $\boxed{(B)}$.

EXAMPLE 2: Line m passes through the points $(k,7)$ and $(3,k-4)$ in the xy-plane. If the slope of line m is 3, what is the value of k?

$$\text{Slope} = \frac{(k-4)-7}{3-k} = 3$$

$$k - 11 = 3(3-k)$$
$$k - 11 = 9 - 3k$$
$$4k = 20$$
$$k = \boxed{5}$$

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In addition to slope, you also need to know what x and y intercepts are. The x-intercept is where the graph crosses the x-axis. Likewise, the y-intercept is where the graph crosses the y-axis.

Let's say we have the line defined by

$$2x + 3y = 12$$

To find the x-intercept, set y equal to 0.

$$\begin{aligned} 2x + 3(0) &= 12 \\ 2x &= 12 \\ x &= 6 \end{aligned}$$

The x-intercept is $(6, 0)$.

To find the y-intercept, set x equal to 0.

$$\begin{aligned} 2(0) + 3y &= 12 \\ 3y &= 12 \\ y &= 4 \end{aligned}$$

The y-intercept is $(0, 4)$.

EXAMPLE 3: The function f is defined by $f(x) = 3x + 12$. What is the x-intercept of the graph of $y = f(x)$ in the xy-plane?

A) $(12, 0)$ B) $(4, 0)$ C) $(-4, 0)$ D) $(-12, 0)$

The graph of the function in the xy-plane is defined by the equation $y = 3x + 12$. To find the x-intercept, set y equal to 0:

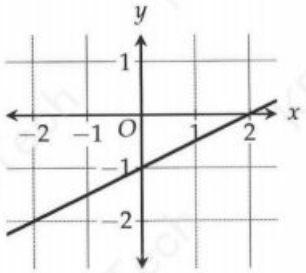
$$\begin{aligned} 0 &= 3x + 12 \\ -12 &= 3x \\ -4 &= x \end{aligned}$$

Therefore, the x-intercept of the graph is $(-4, 0)$, answer $\boxed{(C)}$.

All lines can be expressed in **slope-intercept form**:

$$y = mx + b$$

where m is the slope and b is the y-intercept. So for the line $y = \frac{1}{2}x - 1$, the slope is $\frac{1}{2}$ and the y-intercept is -1:



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While all lines can be expressed in slope-intercept form, sometimes it'll take some work to get there. If you're given a slope and a y-intercept, then of course it's most convenient to express the line in slope-intercept form. But what if we're handed a slope and a point instead of a slope and a y-intercept? Then it'll be more convenient to use **point-slope form**:

$$y - y_1 = m(x - x_1)$$

where (x_1, y_1) is the given point.

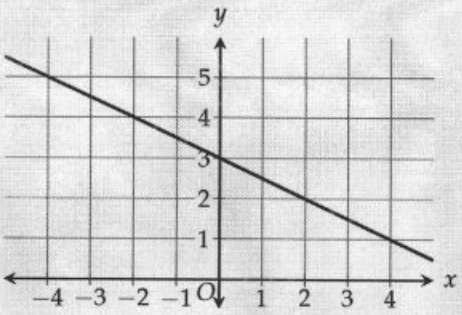
Let's say we want to find the equation of a line that has a slope of 3 and passes through the point $(1, -2)$. The equation of the line in point-slope form is then

$$y - (-2) = 3(x - 1)$$

Once it's in this form, we can manipulate the equation to get the slope-intercept form of the line if needed:

$$\begin{aligned} y - (-2) &= 3(x - 1) \\ y + 2 &= 3x - 3 \\ y &= 3x - 5 \end{aligned}$$

EXAMPLE 4:



What is an equation of the graph shown?

A) $y = -2x + 3$ B) $y = \frac{1}{2}x + 3$ C) $y = -\frac{1}{2}x + 3$ D) $y = 2x - 3$

All the answer choices are in slope-intercept form. To get the equation of the line in slope-intercept form, $y = mx + b$, we need to find the slope m and the y-intercept b. The line crosses the y-axis at 3, so $b = 3$. The line goes down 1 unit for every 2 units to the right, so the slope m is $-\frac{1}{2}$. Therefore, the equation of the line is $y = -\frac{1}{2}x + 3$. Answer $\boxed{(C)}$.

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EXAMPLE 5: Line l passes through the points $(-2,3)$ and $(3,13)$ in the xy-plane. What is the y-coordinate of the y-intercept of line l ?

The slope of line l is $\frac{y_2-y_1}{x_2-x_1}=\frac{13-3}{3-(-2)}=2$. With the given point $(3,13)$, we can use point-slope form to find the equation of the line:

$$y-13=2(x-3)$$

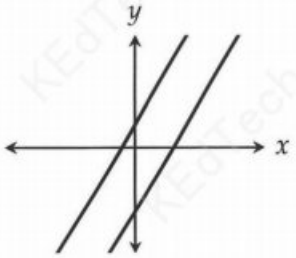
Note that we could've used the other point $(-2,3)$; the result will be the same. The next step is to simplify this equation to get the slope-intercept form of the line:

$$\begin{aligned} y-13&=2(x-3)\\ y&=2x-6+13\\ y&=2x+7 \end{aligned}$$

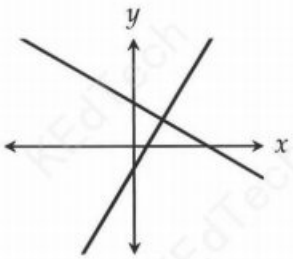
Now we can easily see that the y-coordinate of the y-intercept is $\boxed{7}$.

There are a few more things you need to know about lines.

Two lines are parallel if they have the same slope.



Two lines are perpendicular if the product of their slopes is -1. In other words, if one slope is the negative reciprocal of the other (e.g. 2 and $-\frac{1}{2}$).



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EXAMPLE 6: Line m has a slope of $\frac{2}{3}$ and passes through the point $(4,3)$ in the xy-plane. Line n is perpendicular to line m and passes through the same point $(4,3)$. Which equation defines line n ?

A) $y=-\frac{2}{3}x+9$ B) $y=-\frac{3}{2}x-3$ C) $y=-\frac{3}{2}x+6$ D) $y=-\frac{3}{2}x+9$

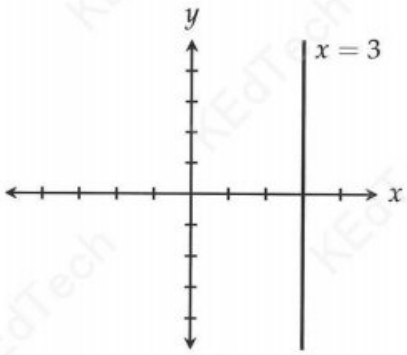
Because it's perpendicular to line m, line n must have a slope of $-\frac{3}{2}$ (the negative reciprocal). Now we can use point-slope form and simplify:

$$y-3=-\frac{3}{2}(x-4)$$
$$y=-\frac{3}{2}x+6+3$$
$$y=-\frac{3}{2}x+9$$

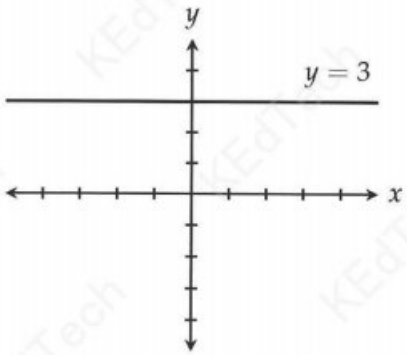
Answer $\boxed{(D)}$.

Finally, let's take a look at the equations of horizontal and vertical lines.

The equation of the vertical line that passes through $(3,0)$ is $x=3$. Every point on the line has an x-coordinate of 3.



The equation of the horizontal line that passes through $(0,3)$ is $y=3$.



Every point on the line has a y-coordinate of 3.

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EXERCISE 1: Answers for this exercise start on page 333.

1

What is the slope of a line that is perpendicular to the graph of $y = -\frac{1}{4}x + 5$ in the xy-plane?

2

What is the x-intercept of the graph of $8x + 12y = 48$ in the xy-plane?

A) $(4, 0)$
B) $(6, 0)$
C) $(8, 0)$
D) $(12, 0)$

3

In the xy-plane, the y-intercept of the graph of $5x - 8y = 40$ is $(0, c)$. What is the value of c ?

4

In the xy-plane, line w has a slope of -8 and passes through the point $\left(\frac{1}{2}, -1\right)$. Which equation defines line w ?

A) $y = \frac{1}{2}x - 1$
B) $y = -8x - 5$
C) $y = -8x + 3$
D) $y = 8x - 5$

5

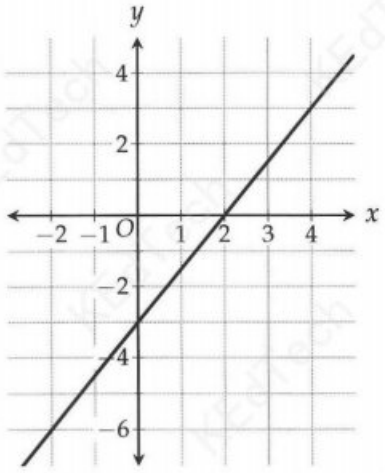
What is the slope of the graph of $y = -11x - \frac{2}{5}(18 - 6x)$ in the xy-plane?

6

In the xy-plane, a line has an x-intercept at $(-2, 0)$ and a y-intercept at $(0, -4)$. What is the slope of the line?

A) -2
B) $-\frac{1}{2}$
C) $\frac{1}{2}$
D) 2

7



The graph of line l is shown in the xy-plane. Which of the following is an equation of a line that is parallel to line l ?

A) $y = -\frac{2}{3}x + 2$
B) $y = \frac{2}{3}x + 10$
C) $y = \frac{3}{2}x - 4$
D) $y = 3x - 1$

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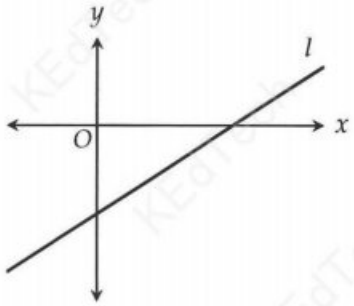
8

Day	Average speed, s (miles per hour)	Number of calories burned, c
Monday	7.2	616
Thursday	6.8	584
Friday	7.9	672
Saturday	8.5	720

On certain days of the week, Elaine runs for an hour on a treadmill. For each day that she ran on the treadmill last week, the table shows the average speed s at which she ran, in miles per hour, and the number of calories c she burned during the run. If the relationship between c and s can be modeled by a linear function, which of the following functions models this relationship?

A) $c(s) = 30s + 400$
B) $c(s) = 60s + 210$
C) $c(s) = 80s + 40$
D) $c(s) = 90s - 30$

9



The graph of line l is shown in the xy-plane. If line l can be represented by the equation $y = mx + b$, where m and b are constants, which of the following must be true?

A) $mb > 0$
B) $mb < 0$
C) $mb = 0$
D) $mb = 1$

10

Line k passes through the point $(1, 5)$ in the xy-plane. If line k is perpendicular to the line with equation $y = \frac{1}{2}x + 3$, which equation defines line k?

A) $y = -2x + 3$
B) $y = -2x + 7$
C) $y = -\frac{1}{2}x + \frac{11}{2}$
D) $y = \frac{1}{2}x + \frac{9}{2}$

11

In the xy-plane, a line with a slope of $\frac{1}{3}$ passes through the points $(-1, 1)$ and $(5, n)$. What is the value of n?

A) 9
B) 4
C) 3
D) $\frac{7}{3}$

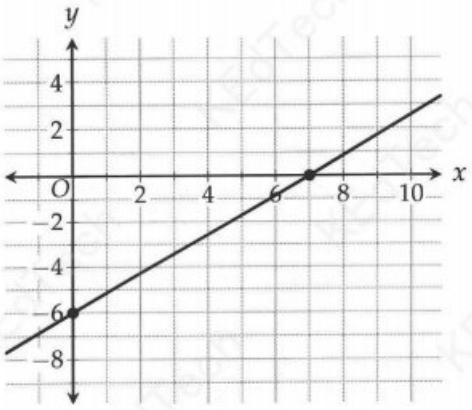
12

x	$f(x)$
-3	10
3	14
9	18

The table shows several values of x and their corresponding values of $f(x)$. There is a linear relationship between x and $f(x)$ that is defined by the equation $f(x) = ax + b$, where a and b are constants. What is the value of ab?

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Line p is shown in the xy-plane, and the point with coordinates $(5, r)$ lies on line p. What is the value of r ?

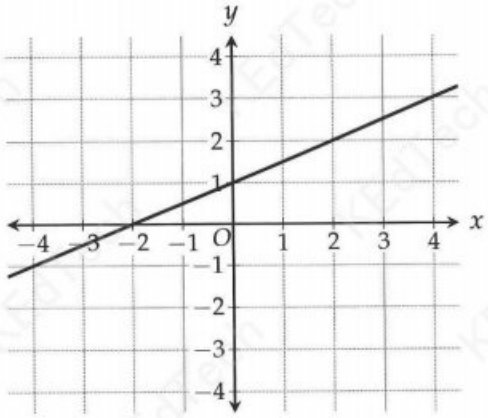
A) $-\frac{5}{3}$

B) $-\frac{11}{6}$

C) $-\frac{11}{7}$

D) $-\frac{12}{7}$

14



Line r is shown in the xy-plane. Line s (not shown) is parallel to line r and passes through the point $(1, -1)$. Which of the following points lies on line s ?

A) $(-3, -3)$

B) $(-2, -2)$

C) $(0, -2)$

D) $(4, 0)$

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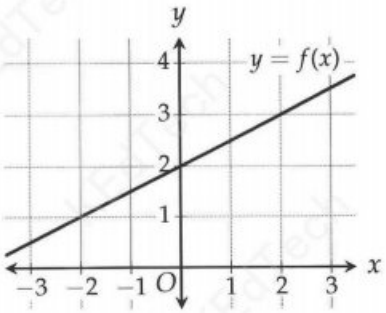
$$6x + y = 17$$
$$hx - 9y = -8$$

In the given pair of equations, h is a constant. The graphs of the equations in the xy-plane are perpendicular lines. What is the value of h ?

16

A line in the xy-plane passes through the point $(6, -3)$ and crosses the x-axis at the point $(15, 0)$. If the line also passes through the point $(-9, t)$, what is the value of t ?

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In the xy-plane, the graph of the linear function f is perpendicular to the graph of the linear function g (not shown). If the graphs of f and g intersect at the point $\left(1, \frac{5}{2}\right)$, what is the value of $g(-1)$?

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