7

Interpreting Linear Equations & Models

On the SAT, you will encounter linear model questions that are a direct extension of our previous chapters on lines and word problems. You'll be asked to figure out the meaning of certain variables and numbers in these models within a real world context.

EXAMPLE 1:

$$V(t) = 240{,}000 - 5{,}000t$$

The given function V models the value, in dollars, of a home from 2006 to 2015, where t is the number of years after 2006. Which of the following is the best interpretation of 240,000 in this context?

A) The value, in dollars, of the home in 2006
B) The value, in dollars, of the home in 2015
C) The average value, in dollars, of the home from 2006 to 2015
D) The increase, in dollars, in the value of the home from 2006 to 2015

We're given a linear function in $y = mx + b$ form. The y-intercept b will typically designate an initial value, the value when $x = 0$. In this case, the y-intercept is 240,000 and it describes the value of the home when $T = 0$, zero years after 2006, which, of course, is 2006. Answer $\boxed{(A)}$.

EXAMPLE 2:

$$V(t) = 240{,}000 - 5{,}000t$$

The given function V models the value, in dollars, of a home from 2006 to 2015, where t is the number of years after 2006. Which of the following is the best interpretation of 5,000 in this context?

A) The number of homes sold each year
B) The yearly decrease, in dollars, in the value of the home
C) The difference, in dollars, between the value of the home in 2006 and in 2015
D) The yearly decrease, in dollars, in the value of the home per square foot

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This time, we're asked to interpret 5,000. Again, we should look at the given function as a linear model expressed in $y = mx + b$ form. In this case, the slope m is $-5{,}000$. Since the slope, by definition, is a rate of change, the increase or decrease in y (value of the home) for each unit increase in x (each year), the number 5,000 can best be interpreted as the amount, in dollars, the value of the home decreases by each year. Answer $\boxed{(B)}$.

It's important that you don't get tricked into choosing a rate that looks right but ultimately doesn't fit the context set by the variable definitions—in this case, the definitions of t and V. Answer A is wrong because we're not dealing with the number of homes sold; we're dealing with the value of a home. Answer D is wrong because the numbers in the equation aren't on a per-square-foot basis. Always be aware of how the variables in the problem are defined.

EXAMPLE 3: Evan spent a total of \$275 on x cups of regular coffee and y cups of specialty coffee. The equation $3x + 5y = 275$ represents this situation. Which of the following is the best interpretation of 5 in this context?

A) The number of cups of regular coffee Evan bought
B) The number of cups of specialty coffee Evan bought
C) The price, in dollars, of one cup of regular coffee
D) The price, in dollars, of one cup of specialty coffee

Although this is an easy question, we're going to use this example to illustrate a technique that's helpful in dealing with these types of questions. The technique involves looking at the units of each term in the equation.

Here, we're given a linear equation of the form $ax + by = c$. The 275 on the right side of the equation represents the 275 dollars that were spent in total. Since the left side of the equation sums to this 275 dollars, the terms $3x$ and $5y$ must each be in dollars for the equation to make sense:

$$3x \text{ dollars} + 5y \text{ dollars} = 275 \text{ dollars}$$

Therefore, $5y$ represents a dollar amount. Since y is defined as the number of cups of specialty coffee, the only way the units work out is if the coefficient 5 represents a dollar amount per cup of specialty coffee.

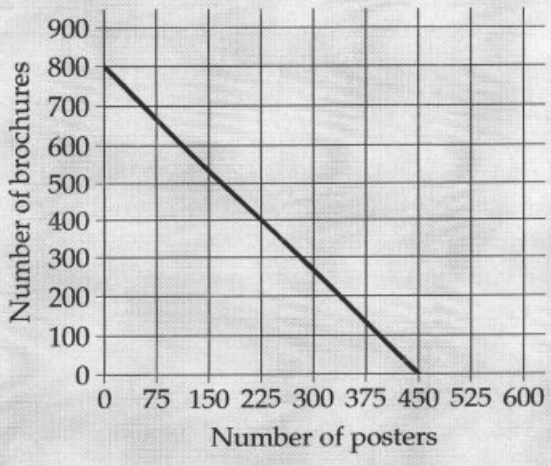
$$5\frac{\text{dollars}}{\text{cup of specialty coffee}} \times y \text{ cups of specialty coffee} = 5y \text{ dollars}$$

Therefore, the only reasonable interpretation of 5 is that it's the price, in dollars, per cup of specialty coffee. Answer $\boxed{(D)}$.

Of course, it's not hard to get to this answer intuitively. It's easy to see that $5y$ must be the amount Evan spent on specialty coffee, so 5 must be the price per cup. There's no need to write out all the units if this is an easy question for you, but this technique may serve you on more difficult questions.

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EXAMPLE 4:



A marketing agency is given a \$7,200 budget to print posters and brochures for a client. The line shown models the possible combinations of posters and brochures that the agency could print. According to the model, what is the cost of printing each poster?

A) \$9
B) \$12
C) \$16
D) \$25

When dealing with a linear model that represents possible combinations, the key is to look at either the x-intercept or y-intercept. Since we want to find the cost of printing each poster, we should look at the x-intercept—450 posters can be printed when no brochures are printed. Therefore, the cost of printing each poster is $\frac{7{,}200}{450} = \$16$. Answer $\boxed{(C)}$.

EXAMPLE 5:

$$h(x) = \frac{4x+6}{5}$$

The function h models the maximum height, in inches, of a plant that was grown using x grams of fertilizer. Based on this model, using 1 additional gram of fertilizer in growing the plant would increase its maximum height by how many inches?

What we're essentially looking for is the change in h (maximum height) for every 1 unit increase in x (fertilizer)—in other words, the slope. To find the slope, we can put the equation into $y = mx + b$ form by splitting up the fraction: $h(x) = \frac{4}{5}x + \frac{6}{5}$. Now we can easily see that the slope is $\frac{4}{5}$ or $\boxed{.8}$. To summarize, the slope of 0.8 indicates that each additional gram of fertilizer increases the maximum height of the plant by 0.8 inches.

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EXAMPLE 6:

$$h(x) = \frac{4x+6}{5}$$

The function h models the maximum height, in inches, of a plant that was grown using x grams of fertilizer. Based on this model, how many more grams of fertilizer should be used in growing the plant to raise the maximum height of the plant by exactly 1 inch?

Because this question is asking for the change in x for every 1 unit increase in h (the reverse of the previous example), we need to rearrange the equation so that we have x in terms of h.

$$\begin{aligned} \frac{4x+6}{5} &= h \\ 4x+6 &= 5h \\ 4x &= 5h-6 \\ x &= \frac{5}{4}h - \frac{3}{2} \end{aligned}$$

The slope of this resulting equation indicates that x must increase by $\frac{5}{4}$, or $\boxed{1.25}$, when we want h to increase by 1.

A shortcut for this type of question is to take the reciprocal of the slope of the original equation: the reciprocal of $\frac{4}{5}$ is $\frac{5}{4}$, or 1.25.

EXAMPLE 7: A can of soda is put into a freezer. The function $T(m) = 65 - 6m$ models the temperature of the soda, in degrees Fahrenheit, m minutes after the can has been put into the freezer. According to the model, what is the decrease in the temperature of the soda, in degrees Fahrenheit, for every 5 minutes the can is left in the freezer?

The slope of -6 represents the change in the temperature of the soda for every 1 minute the can is left in the freezer. So for every 5 minutes left in the freezer, the temperature of the soda decreases by $5 \times 6 = \boxed{30}$ degrees Fahrenheit.

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EXERCISE 1: Answers for this exercise start on page 338.

1

The function $w(x) = 100 - 3x$ gives the water level, in feet, in a large aquarium x days after the aquarium was last refilled. Which statement is the best interpretation of the slope of the graph of $y = w(x)$ in the xy-plane in this context?

A) The water level decreases by 3 feet each day.
B) The water level increases by 3 feet each day.
C) The water level decreases by 100 feet each day.
D) The water level increases by 100 feet each day.

2

Annie collects educational puzzle sets for her classroom. She has B beginner-level puzzle sets and E intermediate-level puzzle sets. There are a total of 1,400 puzzle pieces in all her puzzle sets. The equation $36B + 64E = 1{,}400$ represents this situation. An intermediate-level puzzle set has how many more puzzle pieces than a beginner-level puzzle set?

3

$$f(x) = 24 + 6x$$

The given function f models the weight, in ounces, of a serving tray containing x empty glasses. According to the model, how many ounces does each empty glass weigh?

A) 4
B) 6
C) 12
D) 24

4

A total of 175 ounces of flour was used to make bagels and muffins. The equation $4.8x + 1.5y = 175$ represents this situation, where x is the number of bagels made and y is the number of muffins made. According to the equation, how many ounces of flour are used to make each bagel?

5

A local football team consists of 11 starting players and additional backup players. At the start of the football season, a total of 159 jerseys were purchased for the team, and each player received 3 jerseys. The equation $3(x + 11) = 159$ represents this situation. Which of the following is the best interpretation of the expression $x + 11$ in this context?

A) The number of backup players on the team
B) The number of starting players on the team
C) The number of players on the team
D) The number of jerseys purchased for the starting players on the team

6

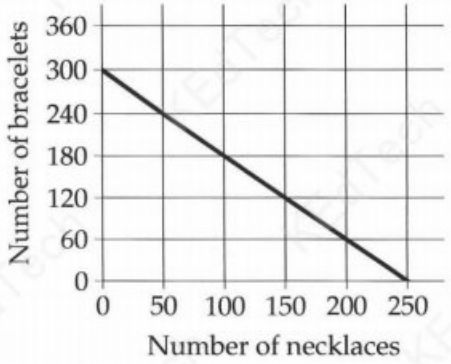
$$y = 177 + 42x$$

The equation gives the number of juniper trees y in a certain forest x years after 2010. What is the best interpretation of $(x, y) = (5, 387)$ in this context?

A) The number of juniper trees in the forest increased by 387 in 2015.
B) The number of juniper trees in the forest increased by 387 from 2010 to 2015.
C) There were 387 juniper trees in the forest in 2005.
D) There were 387 juniper trees in the forest in 2015.

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The line shown models the possible combinations of the number of bracelets and necklaces a certain company with 25 jewelers can handcraft in a month, based on the constant rate at which the jewelers work. Based on this model, what is the maximum number of bracelets that one jeweler at this company could handcraft in a month?

A) 10
B) 12
C) 21
D) 30

8

Ivy bought an 80-ounce block of cheese. She then cut the block into x slices of cheese and y cubes of cheese. The equation $0.6x + 0.25y = 80$ represents this situation. Which is the best interpretation of $0.6x$ in this context?

A) The amount of cheese, in ounces, in each cube
B) The amount of cheese, in ounces, in each slice
C) The total amount of cheese, in ounces, Ivy cut into cubes
D) The total amount of cheese, in ounces, Ivy cut into slices

9

During a road trip, Carl used 12 gallons of fuel driving a minivan and 17 gallons of fuel driving a campervan. He traveled a total distance of 590 miles driving both vehicles. The equation $12x + 17y = 590$ represents this situation. Which is the best interpretation of y in this context?

A) The total number of gallons of fuel Carl used driving the campervan
B) The total number of miles Carl drove the campervan for
C) The fuel efficiency, in miles per gallon, of the campervan
D) The average fuel efficiency, in miles per gallon, of the minivan and the campervan

10

Of the 585 photos in a photo album, 315 are color photos and the rest are black-and-white. The equation $6p + 315 = 585$ describes this situation, where p represents the number of pages that contain black-and-white photos. What is the best interpretation of $6p$ in this context?

A) There are $6p$ black-and-white photos.
B) There are $6p$ photos on each page.
C) There are $6p$ pages in total.
D) There are $6p$ photos in total.

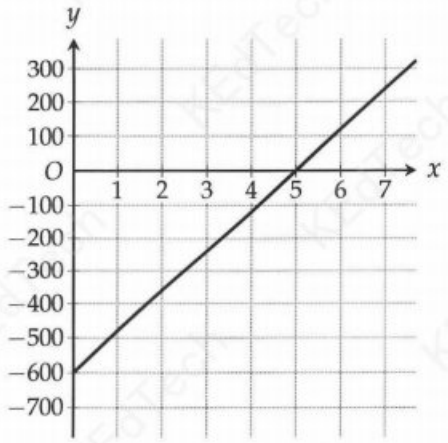
11

Stacy is playing a game in which she needs to collect a certain number of points to win. The function p defined by $p(x) = 52 - 4x$ models the number of additional points Stacy needs to win the game, where x is the number of rounds played. Which of the following is the best interpretation of $p(8) = 20$?

A) Stacy collected a total of 20 points in the first 8 rounds.
B) It will take Stacy 8 rounds to collect the 20 additional points she needs to win the game.
C) After the first 8 rounds, Stacy needs to collect 20 more points to win the game.
D) In the 8th round, Stacy collected 20 more points than she did the previous round.

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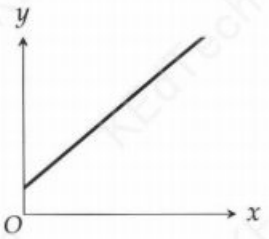
12



The graph models the daily profit y, in dollars, of a bakery after x cakes are sold. Which of the following is the best interpretation of the y-intercept of the graph in this context?

A) The price of each cake
B) The cost of making each cake
C) The daily cost of running the bakery
D) The daily cost of making the cakes that weren't able to be sold

13



A train began its route a certain distance away from a train depot and traveled in a straight path directly away from it. The line shown represents the train's distance from the depot, in miles, x hours after it began its route. What is the best interpretation of the slope of the line?

A) The speed of the train, in miles per hour, during its route
B) The distance, in miles, between the train depot and the train at the start of its route
C) The total distance, in miles, the train has traveled x hours after it began its route
D) The number of hours it took for the train to finish its route

14

$$T = 56 + 5h$$

The given equation models the temperature T of a room, in degrees Fahrenheit, h hours after a heater is turned on. According to the model, the temperature of the heated room increases by how many degrees Fahrenheit every 30 minutes?

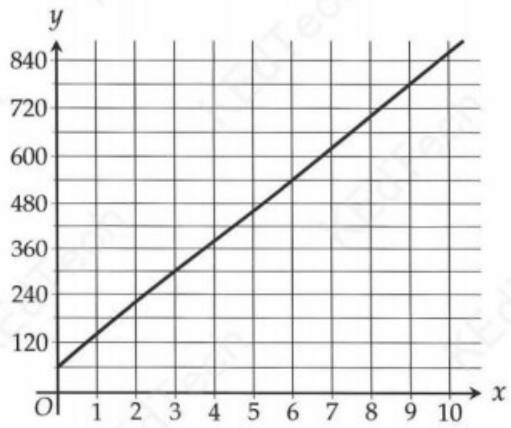
15

A recipe suggests sweetening honey tea with sugar according to the equation $s = 10 - 2h$, where s is the amount of sugar, in teaspoons, that should be added to a tea beverage containing h teaspoons of honey. What is the best interpretation of 2 in this context?

A) For every teaspoon of honey in the beverage, two more teaspoons of sugar should be added.
B) For every teaspoon of honey in the beverage, two fewer teaspoons of sugar should be added.
C) For every two teaspoons of honey in the beverage, one more teaspoon of sugar should be added.
D) For every two teaspoons of honey in the beverage, one fewer teaspoon of sugar should be added.

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The graph represents the total time y, in minutes, a lawyer spent reviewing a case and providing x legal consultations. What is the best interpretation of the slope of the graph in this context?

A) The lawyer spent 60 minutes reviewing the case.

B) Each legal consultation lasted 60 minutes.

C) The lawyer spent 80 minutes reviewing the case.

D) Each legal consultation lasted 80 minutes.

17

$$T(x) = \frac{2x + 9}{5}$$

The function T models the time, in seconds, it takes to load a web page with x images. According to the model, by how many seconds does each image increase the load time of a web page?

18

Jamil reached the end of a 26-mile track by walking at a speed of 5 miles per hour and running at a faster speed than he walked. The equation $5t + 9(4 - t) = 26$ represents this situation, where t is the total amount of time, in hours, Jamil spent walking. Which of the following is the best interpretation of 4 in this context?

A) The total distance, in miles, that Jamil ran

B) The speed at which Jamil ran, in miles per hour

C) The total number of hours Jamil spent running

D) The total number of hours Jamil spent walking and running

19

$$A = \frac{7}{2}(9 + x)$$

The function A gives the area, in square inches, of a trapezoid with a height of 7 inches, a shorter base of 9 inches, and a longer base of x inches. If the length of the longer base is increased by 5 inches, by how many square inches does the area of the trapezoid increase?

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