# 8 Percent

**EXAMPLE 1:** Among 10th graders at a school, 40% of the students are Red Sox fans. Among those Red Sox fans, 20% are also Celtics fans. What percent of the 10th graders at the school are both Red Sox fans and Celtics fans?

A) 8% B) 12% C) 20% D) 30%

In percent questions, the word "of" should be translated as multiplication. If we let the number of 10th graders be $x$, then the number of Red Sox fans is $40\% \times x = 0.40x$, and the number of Celtics fans is $0.20(0.40x) = 0.08x$, which means 8% of the 10th graders at the school are fans of both teams. Answer $\boxed{(A)}$.

If you're not as comfortable working with variables, a common strategy in percent questions is to make up a number to represent the total, typically 100. Let's suppose that the number of 10th graders at the school is 100 and see if we get the same answer.

$$\text{Red Sox fans} = 40\% \text{ of } 100 = 40$$
$$\text{Celtics \& Red Sox fans} = 20\% \text{ of } 40 = 8$$

Finally, $\dfrac{8}{\text{Number of 10th graders}} = \dfrac{8}{100} = 8\%$, which is indeed the same answer.

**EXAMPLE 2:** The number $y$ is 18% greater than the number $x$, where $x > 0$. Which expression represents $y$ in terms of $x$ ?

A) $118x$ B) $18x$ C) $1.18x$ D) $0.18x$

The number $y$ is the number $x$ increased by 18%:

$$y = x + 0.18x = (1 + 0.18)x = 1.18x$$

Answer $\boxed{(C)}$. This example can be generalized. To perform any **percent increase** (or "percent greater than"), we multiply the starting value by one plus the percentage (converted to a fraction/decimal). Why the "one plus"? Because a percent increase is the starting value plus 18% on top. So to increase a value by 20%, we multiply that value by 1.20. To increase a value by 44%, we multiply that value by 1.44, and so on.



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**EXAMPLE 3:** The number $y$ is 35% less than the number of $x$, where $x > 0$. Which expression represents $y$ in terms of $x$ ?

A) $0.35x$ B) $0.65x$ C) $\frac{x}{1.35}$ D) $\frac{x}{1.65}$

This is a percent decrease problem. The number $y$ is the number $x$ decreased by 35%:

$$y = x - 0.35x = (1 - 0.35)x = 0.65x$$

Answer $\boxed{(B)}$. To perform any **percent decrease** (or "percent less than"), we multiply the starting value by one minus the percentage (converted to a fraction/decimal). Why the "one minus"? Because it gives us the remaining portion. So to decrease a value by 15%, we multiply by $1 - 0.15 = 0.85$ because 0.85 of that value remains. To decrease a value by 60%, we multiply by $1 - 0.60 = 0.40$ because 0.40 of that value remains, and so on.

**EXAMPLE 4:** A clothing store sells a certain jacket for 20% more than its suggested retail price. Using a rewards card, a customer is able to purchase the jacket for 25% less than what the store sells it for. The customer's purchase price is what percent of the jacket's suggested retail price?

A) 80% B) 84% C) 90% D) 95%

To summarize the question, we essentially have a price increase of 20% and then a decrease of 25%. We'll let the original retail price of the jacket be $p$. The store's price is then $1.20p$ and the customer's purchase price is

$$1.20p \times 0.75 = 1.20 \times 0.75 \times p = 0.90p$$

This $0.90p$ result means that the final purchase price is 90% of the original price (suggested retail price).

Answer $\boxed{(C)}$. This example shows that percent changes can be "chained" and calculated all at once to get the final result:

$$\text{20\% increase, then 25\% decrease} \Rightarrow 1.20 \times 0.75p$$

And let's imagine that the customer had to pay a sales tax of 5% on the final price. We could "chain" that on as well

$$1.20 \times 0.75 \times 1.05p$$

and get the final price without calculating any intermediate values.

The mistake students make is going one step at a time. So if the jacket costs \$100, they'll take 20% of \$100 to get \$20 and add that back to the \$100 to get \$120. Then they'll take 25% of \$120 to get \$30 and subtract it from the \$120. This process is tedious and error-prone.

**For percent questions, don't calculate what the actual changes or intermediate values are at each step. Get directly to the end result by taking the product of the corresponding multiplication factors.**

**EXAMPLE 5:** On Monday, Julie received 240 emails. On Tuesday, the number of emails Julie received was $p$% less than the number she received on Monday. Which expression represents the number of emails Julie received on Tuesday?

A) $240(1 - p)$ B) $240(p - 1)$ C) $240\left(1 - \frac{p}{100}\right)$ D) $240\left(\frac{p}{100} - 1\right)$



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As we've already stated, to find a value that is $p\%$ less than some starting value, we multiply that starting value by one minus $p\%$ (converted to a fraction/decimal).

So to find the value that is $p\%$ less than 240, we first represent $p\%$ as $\frac{p}{100}$ and subtract it from one: $1 - \frac{p}{100}$.

Then we multiply 240 by this factor: $240\left(1 - \frac{p}{100}\right)$. Answer $\boxed{(C)}$.

Don't be fooled by trap answers such as $240(1 - p)$. If $p$ were 25, this expression would result in an unexpected negative value. Always remember that by definition, $p\% = \frac{p}{100}$ and that this fraction/decimal is often what you want to be working with, not the whole number value of $p$ itself.

Let's summarize what we've learned so far.

| | **Translates to ...** | **Example** |
|---|---|---|
| $p\%$ of $x$ | $\frac{p}{100} \times x$ | 15% of $x \Rightarrow \frac{15}{100} \times x = 0.15x$ |
| $p\%$ greater than $x$ | $\left(1 + \frac{p}{100}\right)x$ | 15% greater than $x \Rightarrow \left(1 + \frac{15}{100}\right)x = 1.15x$ |
| $p\%$ less than $x$ | $\left(1 - \frac{p}{100}\right)x$ | 15% less than $x \Rightarrow \left(1 - \frac{15}{100}\right)x = 0.85x$ |

You should also be able to translate in reverse as well. So when you see $1.15x$, that's equivalent to 15% greater than $x$, and when you see $0.85x$, that's equivalent to 15% less than $x$.

**EXAMPLE 6:** The value of $x$ is $\frac{27}{20}$ times the value of $y$, where $y$ is a positive number. The value of $x$ is what percent greater than the value of $y$ ?

A) 26% B) 35% C) 65% D) 74%

Since $x = \frac{27}{20}y = 1.35y$, $x$ is 35% greater than $y$. Answer $\boxed{(B)}$.

**EXAMPLE 7:** If the number $m$ is 80% less than the number $n$, and the number $n$ is 250% greater than 84, what is the value of $m$ ?

$$n = 84\left(1 + \frac{250}{100}\right) = 84(1 + 2.5) = 294$$

$$m = 294(1 - 0.80) = 294(0.20) = \boxed{58.8}$$

The SAT will often give you percentages you don't see very often, such as "80% less than" or "250% greater than," to trick you into multiplying by 0.80 or 2.5. Don't fall for it! A percent increase requires that we add one to account for the original number. A percent decrease requires that we subtract from one to get the remaining portion.



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**EXAMPLE 8:** The chickens on a farm laid 30% more eggs this year than they did last year. If they laid 5,200 eggs this year, how many did they lay last year?

$x(1+\frac{30}{100})=5200$

$1.3x=5200$

$x=4000$

To summarize, this year's number is 30% more than last year's number:

$$\text{Last Year (?)} \xrightarrow{\text{30\% increase}} \text{This Year (5,200)}$$

Since we're given the number of eggs for this year, the temptation is to decrease this year's number by 30% to get last year's number (to "reverse" the percent change, in other words):

$$\text{Last Year (?)} \xleftarrow{\text{30\% decrease}} \text{This Year (5,200)}$$

Doing so would yield $5{,}200(1-0.30)=5{,}200(0.70)=3{,}640$. However, this answer is incorrect and illustrates a common mistake that students make. We cannot simply "reverse" a percent change this way because percent change depends on the starting value. A 30% increase from last year is not equal in measure to a 30% decrease from this year because the 30% increase is based off of last year's value while the 30% decrease is based off of this year's value.

As a quick example, if we start with 100 and increase it by 50%, we get 150. But if we take this 150 and decrease it by the same percentage, 50%, we get 75, not the original 100 we started with.

So to solve this problem, we need to set up an equation. A 30% increase from last year means that we can multiply last year's number by 1.30 to get this year's number:

$$\begin{aligned}\text{This Year} &= (1.30)(\text{Last Year})\\ 5{,}200 &= (1.30)(\text{Last Year})\\ \frac{5{,}200}{1.30} &= \text{Last Year}\\ \boxed{4{,}000} &= \text{Last Year}\end{aligned}$$

**Remember:** If $x$ is 50% greater than $y$, that does not mean $y$ is 50% less than $x$. When in doubt, set up an equation!

**EXAMPLE 9:** The number of students at a school decreased 20% from 2010 to 2011. If the number of students enrolled in 2011 was $k$, which of the following expresses the number of students enrolled in 2010 in terms of $k$ ?

A) $0.75k$ B) $1.20k$ C) $1.25k$ D) $1.5k$

This example illustrates the same concept as Example 8 but with variables instead of numbers. The answer is NOT $1.20k$. Again, we need to set up an equation. If $x$ is the number of students in 2010, then

$$\begin{aligned}0.80x &= k\\ x &= \frac{1}{0.80}k\\ x &= 1.25k\end{aligned}$$

Therefore, there were 25% more students in 2010 than in 2011. Answer $\boxed{(C)}$.



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So far, we've dealt with problems where the percent change is given to us. In problems where we have to calculate the percent change, we use the following formula:

$$\text{Percent change} = \frac{\text{new value} - \text{original value}}{\text{original value}} \times 100\%$$

For example, if the price of a dress increases from 80 dollars to 90 dollars, the percent change is:

$$\frac{90-80}{80} \times 100\% = 12.5\%$$

If percent change is positive, it's a percent increase. Negative? Percent decrease. From the formula, we can see that percent change is always based off of the original value, which serves as the denominator.

**EXAMPLE 10:** In a particular store, the number of TVs sold during the first week of November was 685. The number of TVs sold during the second week was 500. The number of TVs sold during the second week was what percent less than the number of TVs sold during the first week (rounded to the nearest percent)?

A) 17% B) 27% C) 37% D) 47%

For percent change questions worded as "percent greater than" or "percent less than," the quantity that comes after the word "than" should be considered the "original value" in the context of the percent change formula because it's the quantity being compared against. In this case, the number of TVs sold during the first week is the basis for comparison and should be used as the "original value":

$$\text{Percent change} = \frac{\text{new value} - \text{original value}}{\text{original value}} \times 100\% = \frac{500-685}{685} \times 100\% \approx -27\%$$

Answer $\boxed{(B)}$.

**EXAMPLE 11:** 200 is $x$% greater than 50. What is the value of $x$ ?

It's tempting to answer 400% since 200 is 4 times 50, but this answer is wrong. It's true that 200 is 400% *of* 50, but 200 is not 400% *greater than* 50 (400% greater than 50 is actually 50(1 + 4) = 250).

It's best to use the formula:

$$\frac{\text{new value} - \text{original value}}{\text{original value}} \times 100\% = \frac{200-50}{50} \times 100\% = 3 \times 100\% = 300\%$$

Therefore, $x = \boxed{300}$.



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**EXERCISE 1:** Answers for this exercise start on page 341.

**1**

280 is 32% of what number?

875

**2**

If $n$% of 40 is 9.6, what is $n$% of 250 ?

A) 45
B) 60
C) 75
D) 80

**3**

Approximately 15% of a town's residents live within the town's downtown area. If the town has a total of $r$ residents, which expression best represents the number of residents who live outside the downtown area?

A) $0.15r$
B) $0.85r$
C) $1.15r$
D) $6.67r$

**4**

The result of decreasing a positive number $x$ by $p$% can be represented by the expression $0.12x$. What is the value of $p$ ?

A) 0.12
B) 0.88
C) 12
D) 88

**5**

A factory's electricity consumption increased by 2% from 2022 to 2023. If the factory's electricity consumption in 2023 is $n$ times its electricity consumption in 2022, what is the value of $n$ ?

A) 0.02
B) 0.98
C) 1.02
D) 1.2

**6**

What is the result of increasing 150 by 300%?

$150 \times (100\% + 300\%) = 150 \times 4 = 600$

**7**

In 2019, the population of Lisbon, Portugal was about 2.3 million. The population of Lisbon in 2019 was about 28% of the total population of Portugal. Which of the following is the best estimate of the total population of Portugal in 2019?

A) 1.8 million
B) 3.2 million
C) 8.2 million
D) 12.2 million

**8**

Joanne used a coupon to receive a 20% discount on the purchase of a book. She paid $29.00 after the coupon was applied. What was the price, in dollars, of the book before the coupon was applied?

36.25

$0.8x = 29 \quad x = 36.25$



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**9**

90 is $u$% less than 450. What is the value of $u$ ?

800

**10**

The number of dishes served by a restaurant during dinner was 17.5% greater than the number of dishes served during lunch. If the restaurant served 940 dishes during dinner, how many more dishes did the restaurant serve during dinner than during lunch?

$940 = (1+\frac{17.5}{100})x$ $x = 800$

$940 - 800 = 140$

**11**

Sims spent 76% more on groceries in 2016 than in 2015, and she spent 145% more on groceries in 2017 than in 2016. If Sims spent $p$ times as much on groceries in 2017 as in 2015, what is the value of $p$ ?

A) 1.102
B) 1.862
C) 2.552
D) 4.312

**12**

The number $p$ is 60% of the positive number $q$, and $p$ is 390% of the number $r$. If $q$ is $x$% of $r$, what is the value of $x$ ?

$p = 60\%\, q$

$p = 390\%\, r$

$60\%\, q = 390\%\, r$ $x = 650$

$q = \frac{3.9}{0.6} r = 6.5r$

**13**

Of the clothing items in a store, 24% are vests. Of the vests, 36% are blue. Of the blue vests, 78% are fleece. Which of the following is closest to the percentage of the clothing items in the store that are not blue fleece vests?

A) 10.70%
B) 67.39%
C) 89.30%
D) 93.26%

$24\% \times 36\% \times 78\% = 0.067392 \Rightarrow 6.7\%$

$100\% - 6.7\% = 93.3\%$

**14**

The volume of air in a balloon at the start of an experiment is 258% of the volume of air in the balloon at the end of the experiment. If the volume of air in the balloon at the end of the experiment is $n$% of the volume of air in the balloon at the start of the experiment, what is the value of $n$, to the nearest whole number?

$V_{start} = 258\% V_{end} = 2.58 V_{end}$

$\frac{1}{2.58} V_{start} = V_{end}$ $\frac{1}{2.58} \approx 0.39$

$n = 39$ $= 39\%$

**15**

In 2016, County A and County B collected the same amount of taxes. In 2017, the amount of taxes collected by County A decreased by 25% and the amount of taxes collected by County B increased by 20%. If County A collected 60 million dollars of taxes in 2017, what was the amount of taxes, in millions of dollars, County B collected in 2017?

A) 54
B) 78
C) 90
D) 96



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