10

Rates, Conversions, & Ratios

The best way to approach most questions involving rates is to use conversion factors (if you've gone through high school chemistry, you should know what I'm talking about). Conversion factors are multipliers that help you go from one set of units to another. For example, $\frac{12 \text{ months}}{1 \text{ year}}$ is a conversion factor that converts years into months. Not only can they represent standard unit conversions (e.g. $\frac{60 \text{ minutes}}{1 \text{ hour}}$), they can also represent information provided by the question (e.g. $\frac{1 \text{ ticket}}{50 \text{ dollars}}$). When using conversion factors to solve a problem, you must set them up in the right sequence and with the appropriate numerators and denominators.

EXAMPLE 1: A bicycle manufacturer can produce 20 bicycles per hour. How many hours would it take the manufacturer to produce 320 bicycles?

Intuitive Approach: We divide the total by the rate to get $320 \div 20 = \boxed{16}$ hours. Easy.

Conversion Factor Approach: The given rate of 20 bicycles per hour can be used as a conversion factor: either $\frac{20 \text{ bicycles}}{1 \text{ hour}}$ or its reciprocal, $\frac{1 \text{ hour}}{20 \text{ bicycles}}$. In this case, we want to use the conversion factor $\frac{1 \text{ hour}}{20 \text{ bicycles}}$ so that when we multiply by the given 320 bicycles, the "bicycles" unit cancels out and the "hours" unit is what remains:

$$320 \cancel{\text{ bicycles}} \times \frac{1 \text{ hour}}{20 \cancel{\text{ bicycles}}} = \frac{320 \text{ hours}}{20} = \boxed{16} \text{ hours}$$

Having the correct unit remaining at the end confirms that we used the correct conversion factor. If the question had asked us to convert a given number of hours into the number of bicycles that can be made, we would've used the other conversion factor, $\frac{20 \text{ bicycles}}{1 \text{ hour}}$.

Although the conversion factor approach seems tedious, I recommend using this approach by default to keep your work organized and to avoid silly mistakes, especially for the harder problems. Ultimately, you are using conversion factors in some form or fashion whether you reason out the conversions in your head or write them out explicitly.

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EXAMPLE 2: A box at the supermarket can hold 6 oranges each. Each orange costs 20 cents. Given that the supermarket has a budget of \$540 to stock oranges, how many boxes will the supermarket be able to fill?

Intuitive Approach: If each orange is 20 cents, then a dollar would be enough for 5 oranges. Five hundred forty dollars would then be enough for $540 \times 5 = 2700$ oranges, which would fill $2700 \div 6 = \boxed{450}$ boxes.

Conversion Factor Approach: Typically, our setup will begin with a starting constraint given by the question (in this case, 540 dollars) or a target that needs to be reached (e.g. 320 bicycles in Example 1).

$$540 \text{ dollars} \times \frac{100 \text{ cents}}{1 \text{ dollar}} \times \frac{1 \text{ orange}}{20 \text{ cents}} \times \frac{1 \text{ box}}{6 \text{ oranges}} = \frac{540 \times 100 \text{ boxes}}{20 \times 6} = \boxed{450} \text{ boxes}$$

Notice how the units cancel as we go along. If the units are canceling, chances are we're doing things right. And since the answer should be in boxes, the "box(es)" unit that remains confirms that we've indeed done things correctly.

EXAMPLE 3: A car can travel 1 mile in 1 minute and 15 seconds. At this rate, how many miles can the car travel in 1 hour?

$$1 \text{ hour} \times \frac{60 \text{ minutes}}{1 \text{ hour}} \times \frac{60 \text{ seconds}}{1 \text{ minute}} \times \frac{1 \text{ mile}}{75 \text{ seconds}} = \frac{60 \times 60 \text{ miles}}{75} = \boxed{48} \text{ miles}$$

EXAMPLE 4: Tom drives at an average speed of 50 miles per hour. Which expression represents the number of hours it will take Tom to drive $5x$ miles at this speed?

A) $\frac{x}{10}$ B) $\frac{10}{x}$ C) $10x$ D) $250x$

$$5x \text{ miles} \times \frac{1 \text{ hour}}{50 \text{ miles}} = \frac{5x \text{ hours}}{50} = \frac{x}{10} \text{ hours}$$

Answer $\boxed{(A)}$.

EXAMPLE 5: To prepare for class, Mr. Chu has to print a number of booklets with p pages per booklet. If every 5 pages cost c cents to print and he spent a total of d dollars, how many booklets did Mr. Chu print in terms of p, c, and d ?

A) $\frac{cp}{500d}$ B) $\frac{100d}{cp}$ C) $\frac{500d}{cp}$ D) $\frac{5d}{cp}$

$$d \text{ dollars} \times \frac{100 \text{ cents}}{1 \text{ dollar}} \times \frac{5 \text{ pages}}{c \text{ cents}} \times \frac{1 \text{ booklet}}{p \text{ pages}} = \frac{d(100)(5)}{cp} \text{ booklets} = \frac{500d}{cp} \text{ booklets}$$

Answer $\boxed{(C)}$.

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