EXAMPLE 4:

$$M(t) = 400(0.6)^{3t}$$

The function M models the mass, in nanograms, of a particle after t years. Which of the following is the best interpretation of $(0.6)^{3t}$ in this context?

A) The mass of the particle decreases by 60% every 4 months.
B) The mass of the particle decreases by 60% every 3 years.
C) The mass of the particle decreases by 40% every 4 months.
D) The mass of the particle decreases by 40% every 3 years.

With $y = ab^{\frac{t}{k}}$ as a reference, we see that the exponent of the given equation is not in the form of $\frac{t}{k}$, so how are we supposed to find the value of k ? We have to use a little arithmetic trick:

$$M(t) = 400(0.6)^{3t} = 400(0.6)^{t/(1/3)}$$

Multiplying by 3 is the same as dividing by $\frac{1}{3}$. Now we can see that $k = \frac{1}{3}$, which means that the particle's mass changes by a factor of 0.6 every $\frac{1}{3}$ year. That's equivalent to a $1 - 0.6 = 40\%$ decrease every 4 months. Answer $\boxed{(C)}$. Note that this is an example of exponential decay.

EXAMPLE 5: A non-profit organization currently has 50 volunteers. If the organization is able to double the number of volunteers every 8 months, which of the following equations best models the number of volunteers, v, the organization will have t months from now?

A) $v = 50(2)^{\frac{t}{8}}$ B) $v = 50(2)^{\frac{t}{4}}$ C) $v = 50(2)^{t}$ D) $v = 50(2)^{8t}$

Again, we'll use $y = ab^{\frac{t}{k}}$ as a reference. Based on the given information, $a = 50$ and $b = 2$. Since the number of volunteers increases by a factor of 2 every 8 months, and t is in months, $k = 8$. Therefore, the equation that best models the number of volunteers is $v = 50(2)^{\frac{t}{8}}$. Answer $\boxed{(A)}$.

EXAMPLE 6: A non-profit organization currently has 50 volunteers. If the organization is able to double the number of volunteers every 8 months, which of the following equations best models the number of volunteers, v, the organization will have t years from now?

A) $v = 50(2)^{\frac{t}{8}}$ B) $v = 50(2)^{\frac{2t}{3}}$ C) $v = 50(2)^{\frac{3t}{2}}$ D) $v = 50(2)^{8t}$

This question is the same as the one in Example 3, except we now have inconsistent units of time. We again have a growth factor of 2, and since k (from our reference equation) is the amount of time it takes for a single change by the growth factor, we might assume that $k = 8$. However, this answer would be wrong because k is in months when the t in our equation needs to be in years.

To form the correct equation, t and k must be in the same units, so we have to convert 8 months into years: $k = \frac{8}{12} = \frac{2}{3}$ years. The equation is then $v = 50(2)^{t/(2/3)} = 50(2)^{\frac{3t}{2}}$. Answer $\boxed{(C)}$.

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EXAMPLE 7:

$$f(m) = 25.5(1.72)^m$$

The given function f models the temperature of a substance m minutes after it is exposed to sulfur dioxide, where $0 \leq m \leq 10$. Which of the following functions best models the temperature of the substance s seconds after its exposure to sulfur dioxide, where $0 \leq s \leq 600$?

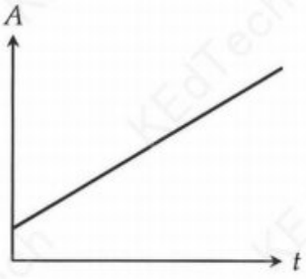
A) $g(s) = 25.5\left(\frac{1.72}{60}\right)^s$ B) $g(s) = 25.5\left(\frac{1.72}{60}\right)^{\frac{s}{60}}$ C) $g(s) = 25.5(1.72)^{\frac{s}{60}}$ D) $g(s) = 25.5(1.72)^{60s}$

The given function f tells us that the temperature of the substance increases by a factor of 1.72 (72% increase) every minute. That's the same as a 1.72 factor increase every 60 seconds. This conversion to seconds is important—because s is in seconds, the "k-value" has to be converted to seconds. So which of the answer choices has a growth factor of 1.72 and a k-value of 60? Only answer $\boxed{(C)}$.

Now let's compare exponential growth and decay with linear growth and decay. As you may already know, **linear growth** can be modeled by a line with a positive slope. For example, if Ann has a piggybank with 100 dollars already in it, and she adds 5 dollars every month, the total amount in the piggybank can be modeled by

$$A = 5t + 100$$

where A is the total amount, t is the number of months, and 100 (the y-intercept) is the initial amount.

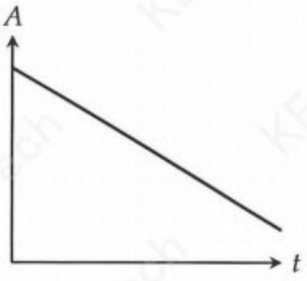


Unlike exponential growth, **linear growth** is constant and consistent. There is no slowing down or speeding up. The total goes up by the same amount each time.

Now imagine that Ann takes 5 dollars out of her piggybank every month instead of adding to it. The total balance would decrease by a constant amount each month, resulting in **linear decay**. The total amount A in the piggybank could then be modeled by

$$A = 100 - 5t$$

The graph of such an equation is a line with a negative slope:



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EXERCISE 1: Answers for this exercise start on page 344.

1

Albert has a large book collection. He decides to trade in two of his used books for one new book each month at a local bookstore. Which of the following best describes the relationship between time (in months) and the total number of books in Albert's collection?

A) Increasing linear
B) Decreasing linear
C) Exponential growth
D) Exponential decay

2

A scientist counts 80 cells in a petri dish and finds that each one splits into two new cells every hour. He uses the function $A(t) = cr^t$ to calculate the total number of cells in the petri dish after t hours. Which of the following assigns the correct values to c and r ?

A) $c = 40, r = 2$
B) $c = 80, r = 0.5$
C) $c = 80, r = 1.5$
D) $c = 80, r = 2$

3

The number of deer in a population is expected to decrease by 11% each year. If the number of deer in the population is currently 12,000, which of the following is closest to the expected number of deer in the population 3 years from now?

A) 8,040
B) 8,460
C) 8,770
D) 9,320

$12{,}000(100\% - 11\%)^3$

4

Two variables, x and y, are related such that the value of y decreases by 25% for every increase in the value of x by 1. If $y = 4$ when $x = 0$, which of the following equations represents this relationship?

A) $y = 0.75(4)^x$
B) $y = 4(0.75)^x$
C) $y = 4(1.25)^x$
D) $y = 16(0.25)^{x+1}$

5

The employees at a new bookstore must stock a certain number of shelves so that the store is ready for its opening in two weeks. The employees stock shelves at a constant rate throughout the two weeks. If $p(t)$ is the number of shelves left to be stocked after t days, which of the following statements best describes the function p ?

A) The function p is an increasing exponential function.
B) The function p is a decreasing exponential function.
C) The function p is an increasing linear function.
D) The function p is a decreasing linear function.

6

The initial value of a car is \$10,000. The function v estimates that the value of the car at the end of each year is 65% of its value at the end of the previous year. Which equation defines v, where $v(k)$ is the estimated value, in dollars, of the car at the end of the kth year?

A) $v(k) = 10{,}000(0.35)^k$
B) $v(k) = 10{,}000(0.65)^k$
C) $v(k) = 10{,}000(1.35)^k$
D) $v(k) = 10{,}000(1.65)^k$

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7

The function $p(t) = 13(1.1)^t$ models the population, in thousands, of a certain town from 2008 to 2020, where t represents the number of years after 2008. Which of the following is the best interpretation of "the value of $p(4)$ is approximately 19" in this context?

A) The population of the town grew by approximately 19 percent each year from 2008 to 2012.

B) The population of the town grew by approximately 19,000 from 2008 to 2012.

C) The population of the town was approximately 19,000 in 2012.

D) The population of the town grew by approximately 19,000 every 4 years between 2008 and 2020.

8

$$f(t) = 20\left(1 + \frac{15}{100}\right)^t$$

The function f models the temperature, in degrees Celsius, of a metal alloy used in an experiment, where t is the number of seconds after the experiment began. Which of the following is the best interpretation of the number 15 in this context?

A) The temperature, in degrees Celsius, of the metal alloy at the beginning of the experiment

B) The increase in the temperature, in degrees Celsius, of the metal alloy every 100 seconds during the experiment

C) The percent by which the temperature, in degrees Celsius, of the metal alloy decreased from each second to the next during the experiment

D) The percent by which the temperature, in degrees Celsius, of the metal alloy increased from each second to the next during the experiment

9

The equation $m(t) = 31.6(1.278)^t$ gives the estimated number of members, in thousands, of a wholesale club for $0 \le t \le 30$, where t is the number of years after 1976. Which of the following is the best interpretation of the number 31.6 in this context?

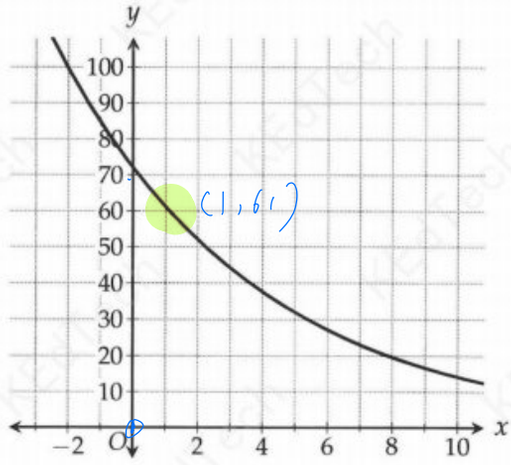
A) The estimated number of members, in thousands, in 1976

B) The estimated number of members, in thousands, t years after 1976

C) The estimated increase in the number of members, in thousands, each year after 1976

D) The estimated percent by which the number of members increased from each year to the next after 1976

10



The graph of $y = f(x)$ is shown, where the function f is defined by $f(x) = a(b)^x$, and a and b are positive constants. Which of the following is closest to the value of $\frac{a}{b}$?

A) 63

B) 85

C) 102

D) 480

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11

Omar currently holds a government bond that has a market value of \$900. Each year, the market value of the bond is expected to be 20% greater than its market value the year before. If the expression $900(1+p)$, where p is a constant, represents the expected market value, in dollars, of the bond after 3 years, what is the value of p ?

0.728

12

Which of the following equivalent forms of the exponential function f displays the value of $f(4)$ as the base or the coefficient?

A) $f(x) = 2\left(\frac{1}{2}\right)^x$

B) $f(x) = 2\left(\frac{1}{4}\right)^{\frac{x}{2}}$

C) $f(x) = 2\left(\frac{1}{8}\right)^{\frac{x}{3}}$

D) $f(x) = 2\left(\frac{1}{16}\right)^{\frac{x}{4}}$

13

The population of trees in a forest has been decreasing by 6 percent every 4 years. The population at the beginning of 2015 was estimated to be 14,000. If P represents the population of trees in the forest t years after 2015, which of the following equations best defines P ?

A) $P = 14{,}000(0.06)^{\frac{t}{4}}$

B) $P = 14{,}000 + 0.94(4t)$

C) $P = 14{,}000(0.94)^{4t}$

D) $P = 14{,}000(0.94)^{\frac{t}{4}}$

14

$$v = 40(1.6)^t$$

The given equation models the resale value of a collectible toy t years after its first release. Which of the following equations best models the resale value, v, of the toy m months after its first release?

A) $v = 40(1.6)^m$

B) $v = 40(1.6)^{\frac{m}{12}}$

C) $v = 40(1.6)^{12m}$

D) $v = 40(1.6)^{\frac{12}{m}}$

15

$$N(h) = 1{,}000(0.97)^{4h}$$

The function N models the number of bacteria in a petri dish after h hours. According to the model, the number of bacteria is predicted to decrease by 3% every k minutes. What is the value of k ?

A) $\frac{1}{4}$

B) 4

C) 15

D) 240

16

The amount of data, in gigabytes, stored in a database increases by 2% every 15 hours. If 16 gigabytes worth of data are currently stored in the database, which of the following functions g gives the amount of data, in gigabytes, that will be stored in the database t days from now?

A) $g(t) = 16(2)^{15t}$

B) $g(t) = 16(1.02)^{\frac{t}{15}}$

C) $g(t) = 16(1.02)^{\frac{5t}{8}}$

D) $g(t) = 16(1.02)^{\frac{8t}{5}}$

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