Long Division of Polynomials

Divide $6x^3 - 19x^2 + 16x - 4$ by $x - 2$, and use the result to factor the polynomial completely.

$$
\begin{array}{r|l}
 & 6x^2 - 7x + 2 \\
\hline
x - 2 & 6x^3 - 19x^2 + 16x - 4 \\
 & \underline{6x^3 - 12x^2} \\
 & -7x^2 + 16x \\
 & \underline{-7x^2 + 14x} \\
 & 2x - 4 \\
 & \underline{2x - 4} \\
 & 0
\end{array}
$$

$-19x^2 - (-12x^2)$

$= -19x^2 + 12x^2$

$= -7x^2$

$$6x^3 - 19x^2 + 16x - 4 = (x - 2)(6x^2 - 7x + 2)$$

$$6x^2 - 7x + 2 = (2x - 1)(3x - 2)$$

$a \quad b \quad c$

$a \cdot c = 6 \cdot 2 = 12$

$a = 6 \quad b = -7 \quad c = 2$

$-3, -4 \qquad (-3)(-4) = 12$

$-3 + (-4) = -7$

$$6x^2 - 3x - 4x + 2 = 3x(2x - 1) - 2(2x - 1)$$

$$= (3x - 2)(2x - 1)$$

$$6x^3 - 19x^2 + 16x - 4 = (x - 2)(3x - 2)(2x - 1) = 0$$

$x = 2, \frac{2}{3}, \frac{1}{2}$

The Division Algorithm

Divide $x^2 + 3x + 5$ by $x + 1$

$$
\begin{array}{r}
x + 2 \\
\hline
x+1 \;\big)\; x^2 + 3x + 5 \\
x^2 + x \\
\hline
2x + 5 \\
2x + 2 \\
\hline
3
\end{array}
$$

$$\frac{x^2 + 3x + 5}{x + 1} = x + 2 + \frac{3}{x+1}$$

$$x^2 + 3x + 5 = (x+1)(x+2) + 3$$

If $f(x)$ and $d(x)$ are polynomials such that $d(x) \neq 0$, and the degree of $d(x)$ is less than or equal to the degree of $f(x)$, then there exist unique polynomials $q(x)$ and $r(x)$ such that

$$f(x) = d(x)\, q(x) + r(x)$$

↓ Dividend ↑ Divisor ↑ Quotient ↑ Remainder

Where $r(x) = 0$ or the degree of $r(x)$ is less than the degree

of $d(x)$. If the remainder is zero, then $d(x)$ divides evenly into $f(x)$

Divide $8x^3 - 1$ by $2x - 1$

$$
\begin{array}{r|l}
2x-1 & 4x^2 + 2x + 1 \\
\hline
 & 8x^3 + 0\cdot x^2 + 0\cdot x - 1 \\
 & \underline{8x^3 - 4x^2} \\
 & 4x^2 + 0\cdot x \\
 & \underline{4x^2 - 2x} \\
 & 2x - 1 \\
 & \underline{2x - 1} \\
 & 0
\end{array}
$$

$0\cdot x^2 - (-4x^2) = 4x^2$

$$\frac{8x^3 - 1}{2x - 1} = 4x^2 + 2x + 1 + \overset{R}{0} \quad x \neq \frac{1}{2}$$

Divide $-2 + 3x - 5x^2 + 4x^3 + 2x^4$ by $x^2 + 2x - 3$

$2x^4 + 4x^3 - 5x^2 + 3x - 2$

$$
\begin{array}{r|l}
x^2 + 2x - 3 & 2x^2 + 1 \\
\hline
 & 2x^4 + 4x^3 - 5x^2 + 3x - 2 \\
 & \underline{2x^4 + 4x^3 - 6x^2}
\end{array}
$$

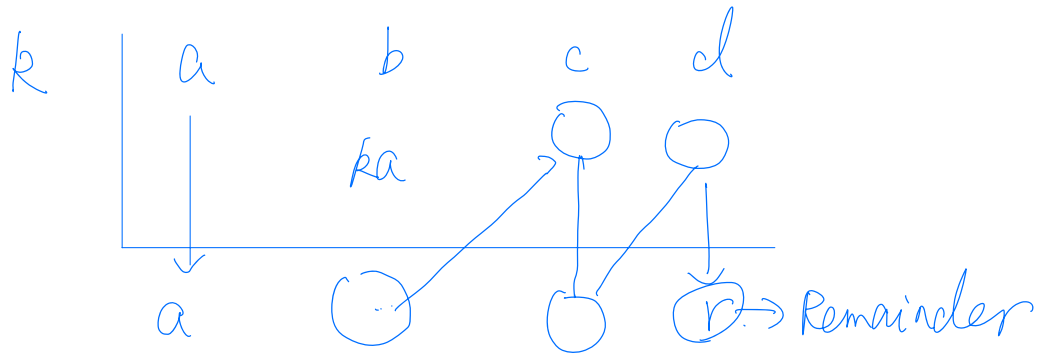
$$x^2+3x-2$$
$$x^2+2x-3$$
$$x+1$$

$$\frac{2x^4+4x^3-5x^2+3x-2}{x^2+2x-3} = 2x^2+1+\frac{x+1}{x^2+2x-3}$$

$$2x^4+4x^3-5x^2+3x-2 = (x^2+2x-3)(2x^2+1)+x+1$$

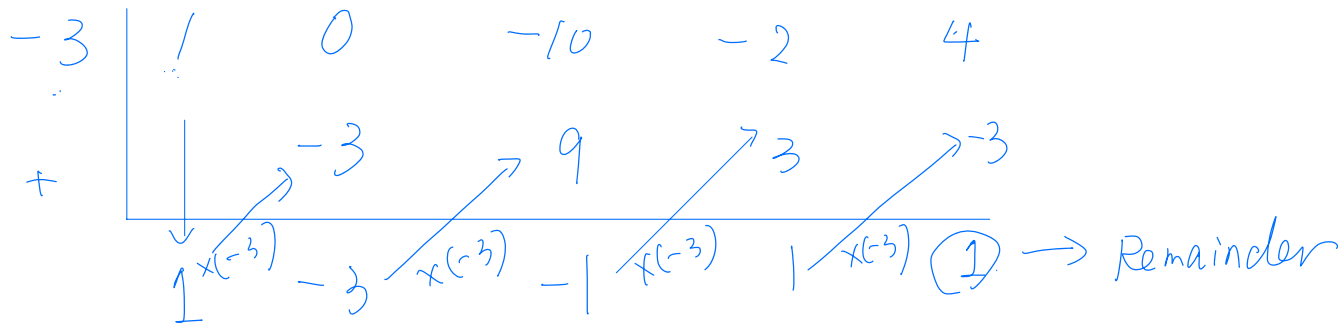
Synthetic Division

Divide ax^3+bx^2+cx+d by $x-k$, use the following pattern.



Divide

x^4-10x^2-2x+4 by $x+3$ $\quad x+3=x-(-3)$



$$\frac{x^4 - 10x^2 - 2x + 4}{x+3} = x^3 - 3x^2 - x + 1 + \frac{1}{x+3}$$

$$x^4 - 10x^2 - 2x + 4 = (x+3)(x^3 - 3x^2 - x + 1) + 1$$

The Remainder and Factor Theorems.

$$f(x) = d(x)\,q(x) + r(x)$$

- $x^2 + 3x + 5 = (x+1)(x+2) + 3$ $\quad x-k \quad x=k$

$x = -1$ $\quad x^2 + 3x + 5 = (-1)^2 + 3(-1) + 5$

$= 1 - 3 + 5 = 3$

Remainder

$x^2 + 3x + 5 = (-1+1)(-1+2) + 3 = 3$

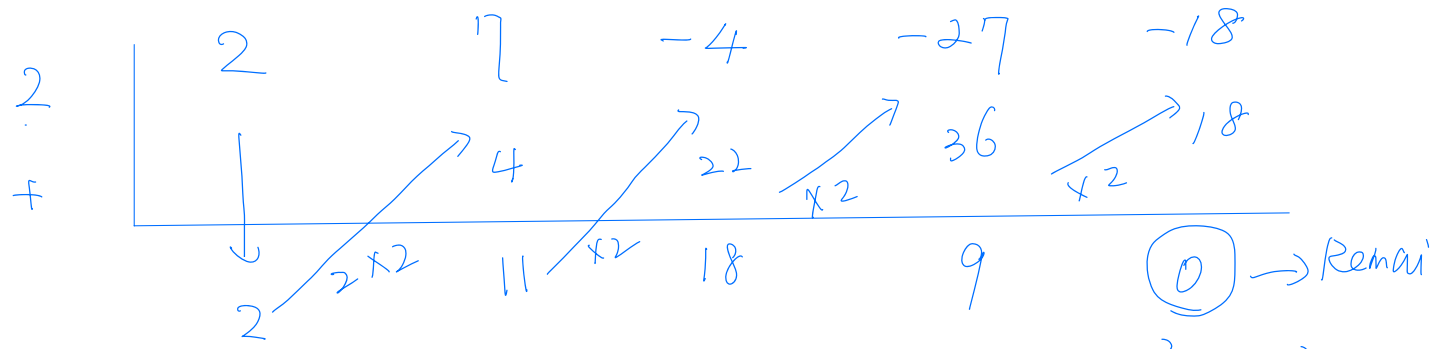
$(-1+1) = 0$

Factoring a Polynomial: Repeated Division

Show that $(x-2)$ and $(x+3)$ are factors of

$$f(x) = 2x^4 + 7x^3 - 4x^2 - 27x - 18$$

Then find the remaining factors of $f(x)$



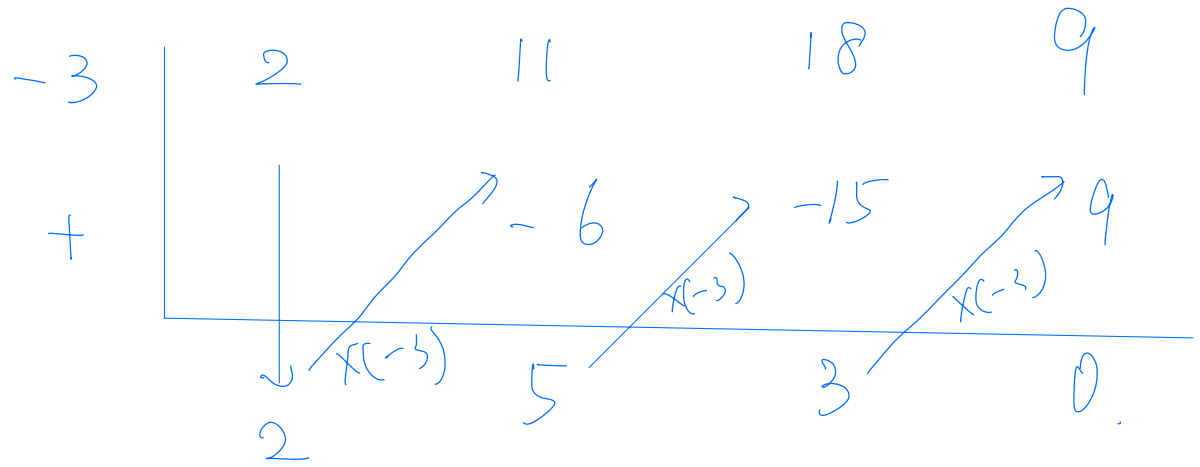
$$2x^4 + 7x^3 - 4x^2 - 27x - 18 = (x-2)(2x^3 + 11x^2 + 18x + 9)$$

$x-(-3)$

$x+3$

$$\begin{array}{r} 2x^3 + x^2 - 7x - 6 \\ \hline x+3 \;\big)\; 2x^4 + 7x^3 - 4x^2 - 27x - 18 \\ 2x^4 + 6x^3 \\ \hline x^3 - 4x^2 \\ x^3 + 3x^2 \\ \hline -7x^2 - 27x \\ -7x^2 - 21x \\ \hline -6x - 18 \\ -6x - 18 \\ \hline 0 \end{array}$$

$$2x^4 + 7x^3 - 4x^2 - 27x - 18 = (x+3)(2x^3 + x^2 - 7x - 6)$$



$$2x^4 + 7x^3 - 4x^2 - 27x - 18 = (x-2)(x+3)(2x^2 + 5x + 3)$$

$2x^2 + 5x + 3$ $\quad a = 2 \quad b = 5 \quad c = 3$

a b c $\quad a \cdot c = 6 \quad 2 + 3 = 5 = b$

$2 \cdot 3 = 6$

$$2x^2 + 2x + 3x + 3 = 2x(x+1) + 3(x+1) = (2x+3)(x+1)$$

$$2x^4 + 7x^3 - 4x^2 - 27x - 18 = (x-2)(x+3)(2x+3)(x+1)$$

The Long Division

Divide $2x^2 + 10x + 12$ by $x + 3$

$$\begin{array}{r} 2x + 4 \\ x+3 \overline{)\,2x^2 + 10x + 12} \\ \underline{2x^2 + 6x} \end{array}$$

$4x+12$

$x-(-3)$ $\quad 4x+12$

0

-3 | 2 $\quad 10$ $\quad 12$

$+$

-6 $\quad 12$

$x(-3)$ $\quad x(-3)$

2 $\quad 4$ $\quad 0$

$2x^2+10x+12=(x+3)(2x+4)$

Divide $5x^2-17x-12$ by $x-4$

$-17x-(-20x)$

$=-17x+20x$

$=3x$

$$\begin{array}{r} 5x+3 \\ x-4 \;\big)\; 5x^2-17x-12 \\ \underline{5x^2-20x} \\ 3x-12 \\ \underline{3x-12} \\ 0 \end{array}$$

$5x^2-17x-12=(x-4)(5x+3)$

4 | 5 $\quad -17$ $\quad -12$

$+$

20 $\quad 12$

5×4 $\quad \times4$

5 $\quad 3$ $\quad 0$

Divide $4x^3-7x^2-11x+5$ by $4x+5$

$-11x-(-15x)$

$=-11x+15x$

$=4x$

$$\begin{array}{r} x^2-3x+1 \\ 4x+5 \;\big)\; 4x^3-7x^2-11x+5 \\ \underline{4x^3+5x^2} \end{array}$$

$$-12x^2 - 11x$$
$$-12x^2 - 15x$$
$$4x + 5$$
$$4x + 5$$
$$0$$

$-\frac{5}{4}$ | 4 | −7 | −11 | 5

+

	−5	15	−5
4	$x(-\frac{5}{4})$	$x(-\frac{5}{4})$	$x(-\frac{5}{4})$
4	−12	4	0

$4x + 5$

$4x - (-5)$

$x - (-\frac{5}{4})$

$$4x^3 - 7x^2 - 11x + 5 = (x + \tfrac{5}{4})(4x^2 - 12x + 4)$$
$$= (x + \tfrac{5}{4}) \cdot 4 (x^2 - 3x + 1)$$
$$= (4x + 5)(x^2 - 3x + 1)$$

Divide $(x + 8 + 6x^3 + 10x^2) \div (2x^2 + 1)$

$6x^3 + 10x^2 + x + 8$

$$2x^2 + 0 \cdot x + 1 \;\overline{)\; 6x^3 + 10x^2 + x + 8}$$
$$3x + 5$$
$$6x^3 + 0 + 3x$$
$$10x^2 - 2x + 8$$
$$10x^2 + 0 + 5$$
$$-2x + 3$$

$$\frac{6x^3+10x^2+x+8}{2x^2+1} = (3x+5) + \frac{3-2x}{2x^2+1}$$