Identify the Vertex of a Quadratic Function

20. $f(x) = 16 - \frac{1}{4}x^2$ $\quad f(x) = a(x-h)^2 + k$

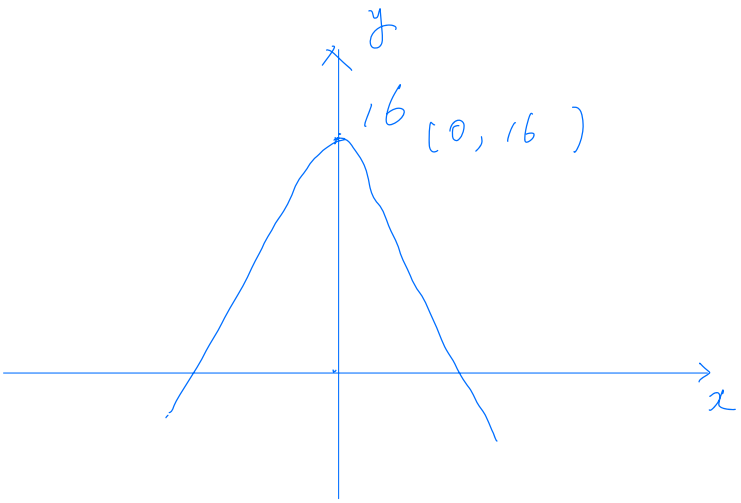
$a = -\frac{1}{4}$ $\quad b = 0$ $\quad f(x) = ax^2 + bx + c$.

$h = -\frac{b}{2a} = -\frac{0}{2\times(-\frac{1}{4})} = 0$

$(0, 16)$

$k = -\frac{1}{4}(0-0)^2 + 16 = 16$

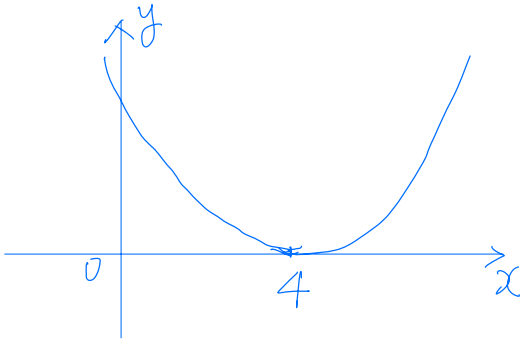
$a < 0$



23. $h(x) = x^2 - 8x + 16$ $\quad a = 1$ $\quad b = -8$ $\quad c = 16$

$-\frac{b}{2a} = -\frac{-8}{2\times 1} = 4$ $\quad h(4) = 4^2 - 8\cdot 4 + 16 = 0$

$h(x) = (x-4)^2$ $\quad a > 0$



$h(x) = x^2 - 8x + 16$

25. $f(x) = x^2 - x + \frac{5}{4}$ $\quad a = 1 \quad b = -1 \quad c = \frac{5}{4}$

$-\frac{b}{2a} = -\frac{-1}{2 \times 1} = \frac{1}{2}$ $\quad f(1) = \left(\frac{1}{2}\right)^2 - \frac{1}{2} + \frac{5}{4}$

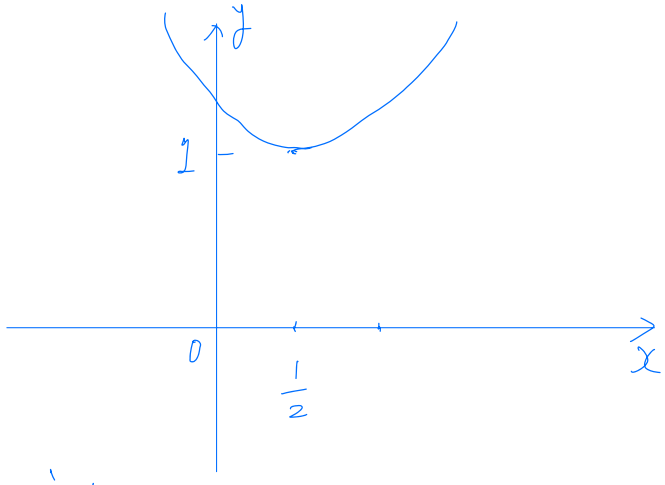
$= \frac{1}{4} - \frac{1}{2} + \frac{5}{4}$

$f(x) = \left(x - \frac{1}{2}\right)^2 + 1$ $\quad = \frac{1 - 2 + 5}{4} = \frac{4}{4} = 1$

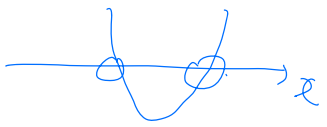
The standard Form:

$f(x) = 1(x - h)^2 + k$ $\quad a \neq 0$ $\quad (h, k)$

$= 1\left(x - \frac{1}{2}\right)^2 + 1$ $\quad a > 0$ $\cup$



Identify x-intercepts



33. $g(x) = x^2 + 8x + 11$ $\quad g(x) = 0 = x^2 + 8x + 11$

$h = -\frac{b}{2a} = -4$ $\quad k = (-4)^2 + 8(-4) + 11 = 16 - 32 + 11 = -5$

$a > 0$ $\cup$



$g(x) = (x + 4)^2 - 5 = 0$

$(x + 4)^2 = 5$ $\quad (-4 + \sqrt{5}, 0)$

$x + 4 = \pm\sqrt{5}$ $\quad (-4 - \sqrt{5}, 0)$

-5

40. Vertex : (4, 1)

point : (6, -7)

$f(x) = a(x-h)^2 + k$

$f(x) = a(x-4)^2 + 1$

$= a(x^2 - 8x + 16) + 1$ ✗

$= ax^2 - 8ax + 16a + 1$

when $x = 6$

$f(6) = a(6-4)^2 + 1 = -7$

$= a \cdot 2^2 + 1 = -7$

$= 4a = -8$

$a = -2$

$f(x) = -2(x-4)^2 + 1$

43. Vertex: $(\frac{1}{2}, 1)$; point : $(-2, -\frac{21}{5})$

$f(x) = a(x - \frac{1}{2})^2 + 1$

$-\frac{21}{5} = a\left[(-2) - \frac{1}{2}\right]^2 + 1$

$-\frac{21}{5} = a\left(-2 - \frac{1}{2}\right)^2 + 1$

$-\frac{21}{5} = a\left(-\frac{4}{2} - \frac{1}{2}\right)^2 + 1$

$-\frac{21}{5} = a\left(-\frac{5}{2}\right)^2 + 1$

$-\frac{21}{5} - 1 = a\frac{25}{4}$

$-\frac{21-5}{5} = a\frac{25}{4}$

$-\frac{26}{5} \times \frac{4}{25} = a$

$a = -\frac{104}{125}$

$f(x) = -\frac{104}{125}(x - \frac{1}{2})^2 + 1$

50. $y = -2x^2 + 10x = -2x(x-5)$

$0 = -2x(x-5)$ $\quad -2x = 0$ or $x - 5 = 0$

$x = 0$ $\quad x = 5$

$a = -2 \quad b = 10 \quad c = 0$

$h = -\frac{b}{2a} = -\frac{10}{2 \cdot (-2)} = \frac{10}{4} = \frac{5}{2}$ $\quad k = -2 \times \left(\frac{5}{2}\right)^2 + 10 \times \frac{5}{2}$

$= -2 \times \frac{25}{4} + 25$

$= -\frac{25}{2} + 25$

$= \frac{50 - 25}{2} = \frac{25}{2}$

$y = -2\left(x - \frac{5}{2}\right)^2 + \frac{25}{2} = 0$

$-2\left(x - \frac{5}{2}\right)^2 = -\frac{25}{2}$

$\left(x - \frac{5}{2}\right)^2 = \frac{25}{4}$

$x - \frac{5}{2} = \pm\frac{5}{2}$

$x = \frac{5}{2} \pm \frac{5}{2}$

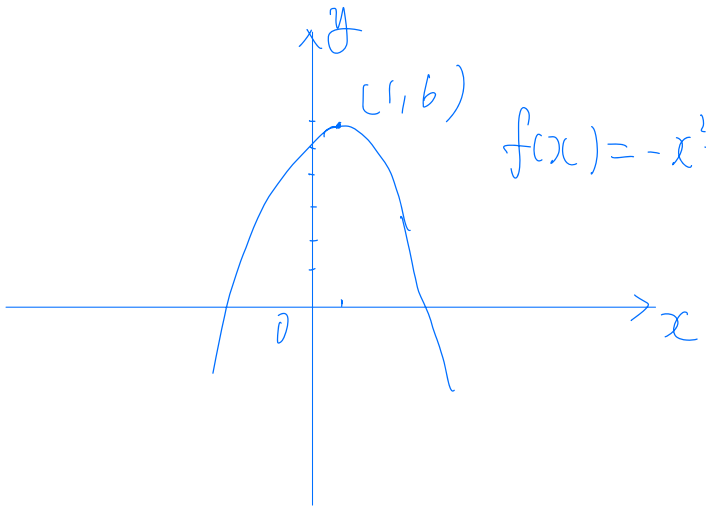
$x = \frac{5}{2} + \frac{5}{2} = 5$ or $x = \frac{5}{2} - \frac{5}{2}$.

27. Graph. vertex

$f(x) = -x^2 + 2x + 5$ $\quad a = -1 \quad b = 2 \quad c = 5$

$h = -\frac{b}{2a} = \frac{-2}{2 \times (-1)} = 1$ ✓ $\quad k = 6$ ✓

$f(x) = -(x-1)^2 + 6$ ✓



$a < 0$

$f(x) = -x^2 + 2x + 5$ ∩

35. $f(x) = -2x^2 + 16x - 31$ $\quad a = -2 \quad b = 16 \quad c = -31$

$h = -\frac{b}{2a} = 4.$ $\quad k = 1$

$f(x) = -2(x-4)^2 + 1$

$0 = -2(x-4)^2 + 1$

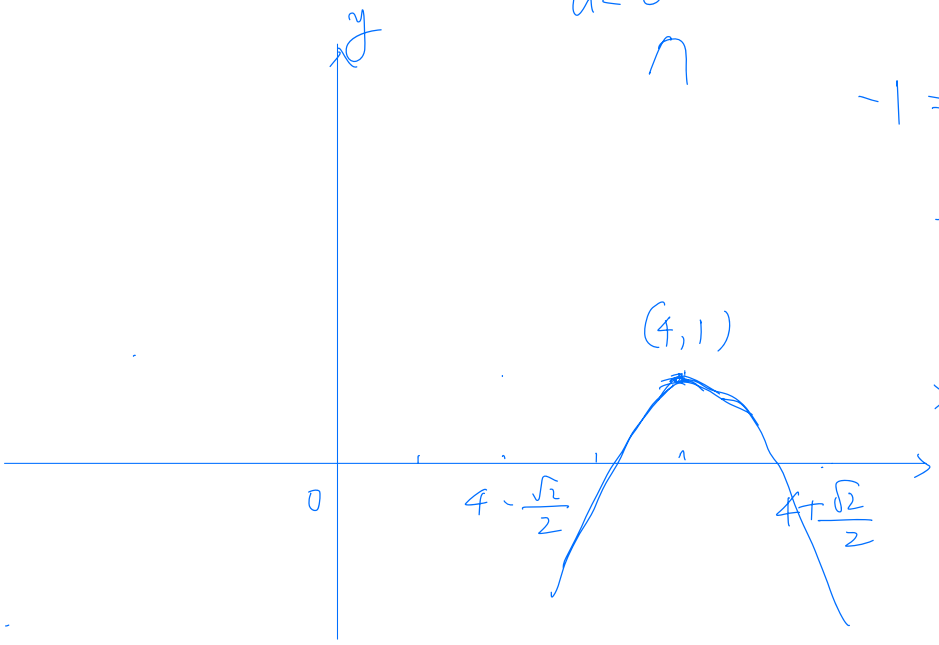
$-1 = -2(x-4)^2$

$\frac{1}{2} = (x-4)^2$

$\pm\frac{\sqrt{2}}{2} = x - 4$

$x = 4 \pm \frac{\sqrt{2}}{2}$

$a < 0$ ∩



$\left(4 + \frac{\sqrt{2}}{2}, 0\right) \quad \left(4 - \frac{\sqrt{2}}{2}, 0\right)$

$\frac{1}{\sqrt{2}} = \frac{1 \cdot \sqrt{2}}{\sqrt{2} \cdot \sqrt{2}} = \frac{\sqrt{2}}{2}$

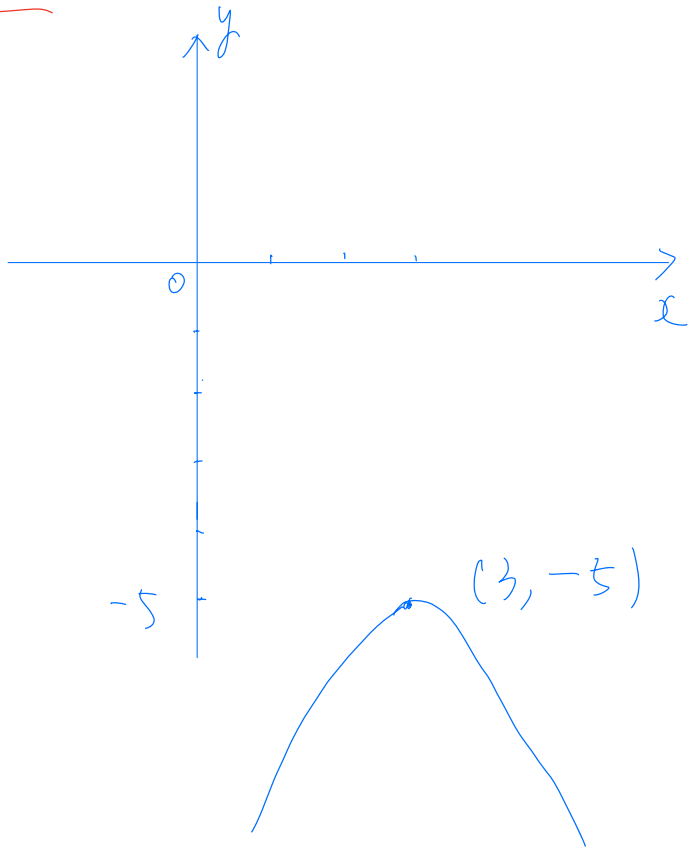
36. $f(x) = -4x^2 + 24x - 41$ $\quad a = -4 \quad b = 24 \quad c = -41$

$h = -\frac{b}{2a} = \frac{-24}{2\times(-4)} = \frac{24}{8} = 3 \qquad k = -5$

$f(x) = -4(x-3)^2 - 5 \qquad 0 = -4(x-3)^2 - 5$

$5 = -4(x-3)^2$

$+ \quad - \quad +$



$a < 0$

∩

41. Vertex : $(1, -2)$; Point : $(-1, 14)$

$\therefore f(x) = a(x-1)^2 - 2$ $\quad$ when $x = -1 \quad f(-1) = 14$

$f(-1) = a(-1-1)^2 - 2 = 14$

$= a(-2)^2 - 2 = 14$

$a = 4$

$f(x) = 4(x-1)^2 - 2$

44. Vertex: $\left(-\frac{1}{4}, -1\right)$; Point: $\left(0, -\frac{17}{16}\right)$

$f(x) = a(x-h)^2 + k$

$f(x) = a\left(x-\left(-\frac{1}{4}\right)\right)^2 - 1 = a\left(x+\frac{1}{4}\right)^2 - 1$

$-\frac{17}{16} = a\left(0+\frac{1}{4}\right)^2 - 1$

$-\frac{17}{16} + 1 = a \times \frac{1}{16}$

$\frac{-17+16}{16} = a \times \frac{1}{16}$ $\quad \frac{-1}{16} = a \times \frac{1}{16}$ $\quad -\frac{1}{16} \times 16 = a$

$a = -1$

$f(x) = -\left(x+\frac{1}{4}\right)^2 - 1$